

A LABORATORY MANUAL OF PHYSICS

FOR ADVANCED LEVEL CERTIFICATE,
SCHOLARSHIP AND INTERMEDIATE SCIENCE STUDENTS

BY
F. TYLER

Heat



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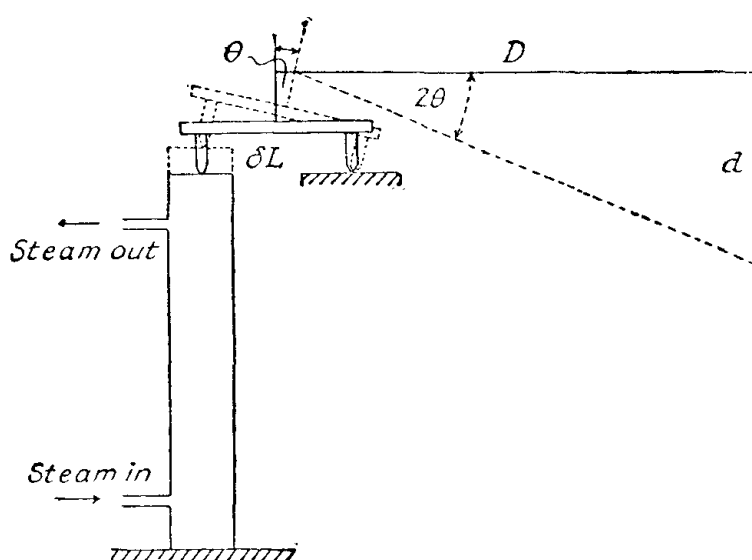
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HEAT

EXPERIMENT 53

DETERMINATION OF THE COEFFICIENT OF LINEAR EXPANSION OF A METAL TUBE USING AN OPTICAL LEVER



Apparatus

A metal tube closed at both ends and fitted with inlet and outlet tubes at the sides near each end for steam circulation, optical lever of standard design, metal block or piece of plate glass firmly clamped in a horizontal position and supported by a heavy stand, lamp and scale, steam heater, thermometer.

Method

The apparatus is fitted up as shown in the diagram with the metal tube supported in a vertical position by clamp and retort stand, and with its lower end on a firm bench. The optical lever is situated with its rear leg on the top of the metal tube and the front two legs on the metal block placed on a level with the top of the tube. The lamp and scale, arranged for vertical displacement, are fixed a measured distance D from the mirror of the lever, and the position of the spot of light on the scale is read. Steam is now passed through the tube and the expansion of the tube causes the mirror to be tilted and produces a deflection of the spot on the scale. The position of the spot is read at its full deflected position. The distance between the front and back legs of the optical lever is now measured, as also is the length of the metal tube when cold. The tube may be assumed to be at air temperature (which is measured by a thermometer placed near it) when cold, and the steam temperature is obtained from tables, having ascertained the atmospheric pressure.

Theory

Let the length of the tube be L cm. and the expansion δL cm. when the temperature is increased from $t^\circ \text{C.}$ to $T^\circ \text{C.}$ Then if b is the distance between the back leg and the line through the two front legs, the angle through which the mirror is tilted is $\frac{\delta L}{b}$ radians.

Now if the deflection of the spot of light on the scale a distance D cm. from the mirror is d cm., the angle through which the beam is deflected is $\frac{d}{D}$ radians. This angle is twice the angle through which the mirror is tilted.

i.e.
$$\frac{d}{D} = 2 \frac{\delta L}{b}$$

or
$$\delta L = \frac{bd}{2D}$$

Thus, since the linear coefficient of expansion (α)

$$= \frac{\text{increase in length}}{\text{original length} \times \text{temperature change}}$$

we have
$$\alpha = \frac{bd}{2DL(T - t)}$$

Results

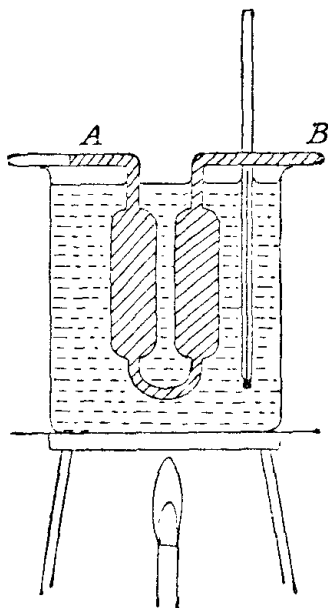
Original length of tube (L)	=	cm.
Room temperature (t)	=	$^\circ \text{C.}$
Steam temperature (T)	=	$^\circ \text{C.}$
Distance (b)	=	cm.
Distance (D)	=	cm.
Deflection (d)	=	cm.

$$\alpha = \underline{\hspace{2cm}} \text{ per } ^\circ \text{C.}$$

Note

Alternatively the experiment can be carried out using a spherometer (see p. 68) to measure the increase in length of the tube. The three legs of the spherometer rest on an ebonite table cut with a central hole to permit of the expansion of the tube. The centre leg is screwed down (through the hole) to make contact with the tube, and the reading taken. It is then screwed back to allow for the expansion, on completion of which a further reading is taken with the centre leg in contact with the tube. In this way the amount of expansion of the tube is obtained from which α is calculated.

DETERMINATION OF THE APPARENT COEFFICIENT OF EXPANSION OF A LIQUID USING A PYKNOMETER



Apparatus

Pyknometer, thermometer (0–100° C.), large beaker.

Method

The pyknometer is first cleaned and dried (by blowing warmed air through it), and weighed. The liquid whose coefficient of expansion is to be measured is then drawn into the pyknometer by sucking at a tube connected to end *B*. By means of blotting-paper, liquid is drawn off at *B* until the pyknometer is full up to the mark *A* as shown. Then the pyknometer is re-weighed by carefully suspending it with cotton from the hook on the balance arm. It is now placed in a beaker containing water whose temperature is slowly raised to some high temperature (say 60° C.). Expansion of the liquid beyond the mark *A* is prevented by absorbing the excess with blotting-paper at *B*. Having maintained a constant high temperature for several minutes, the pyknometer is removed from the water-bath, dried, and again weighed.

Theory

Let d_1 and d_2 be the densities of the liquid at t_1 and t_2 ° C.

Let the volume of the pyknometer from the mark *A* to the end of *B* = V c.c., and let the masses of liquid filling this volume be m_1 and m_2 at the temperatures t_1 and t_2 ° C.

Then $d_1 = \frac{m_1}{V}$ and $d_2 = \frac{m_2}{V}$ (ignoring the expansion of the pyknometer).

Now since the density of a liquid varies inversely as its specific volume, we have—

$$\frac{d_1}{d_2} = \frac{v_2}{v_1} = 1 + C_a(t_2 - t_1) \quad (\text{where } C_a \text{ is the coefficient of apparent expansion of the liquid})$$

i.e. $\frac{m_1}{m_2} = 1 + C_a(t_2 - t_1)$

from which $C_a = \frac{m_1 - m_2}{m_2(t_2 - t_1)} = \frac{\text{weight expelled}}{\text{weight remaining} \times \text{temperature change}}.$

Results

Weight of clean, dry pyknometer (w_1)	=	gm.
„ „ pyknometer full of water at ° C. (w_2)	=	gm.
„ „ „ „ „ „ „ „ ° C. (w_3)	=	gm.

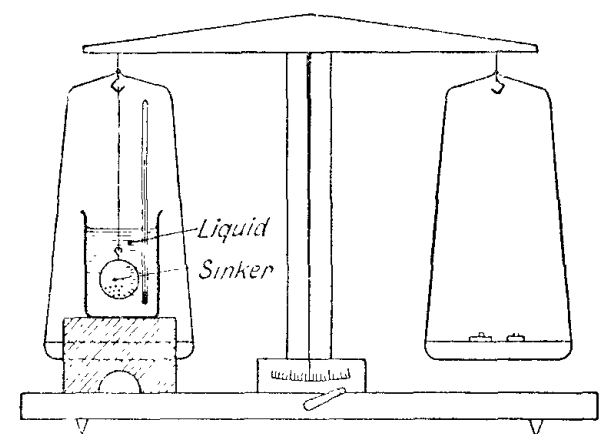
$$\therefore C_a = \frac{\text{weight expelled}}{\text{weight remaining} \times \text{temperature change}} = \frac{w_2 - w_3}{(w_3 - w_1)t_0}$$

$$= \text{per } ^\circ \text{C.}$$

Notes. (1) If a pyknometer is not available, this experiment can be satisfactorily carried out, using an ordinary specific-gravity bottle as the weight thermometer.

(2) Another interesting use of the weight thermometer (a specific-gravity bottle is convenient) is to determine the coefficient of cubical expansion of a solid in the form of a powder. Using a liquid in which the powder is insoluble, the expansion of the liquid against the solid can be found in terms of the relative mass of liquid expelled when the solid is immersed in the liquid in the weight thermometer. Then from previous measurements of the expansion of the liquid when filling the weight thermometer alone, the coefficient of expansion of the powder can be calculated. The details are left to the student.

DETERMINATION OF THE COEFFICIENT OF APPARENT EXPANSION OF A LIQUID BY MATHIESSEN'S SINKER METHOD



Apparatus

Conventional Mathiessen sinker—a glass bulb hermetically sealed and loaded with mercury or lead shot so that it just sinks in the liquid at the low temperature, beam balance and weights, thermometer, Archimedes bridge, beaker containing a supply of the liquid under test, tripod, bunsen.

Method

Weigh the sinker in air. Now re-weigh it when completely immersed in the liquid (see figure) when at room temperature—which is recorded. Now heat the liquid to a temperature not exceeding about 60° C., and re-immers the sinker in the hot liquid for re-weighing. This third weighing can conveniently be obtained by placing weights which are just too light on the right-hand balance pan and, as the liquid cools, noting the temperature at which exact counterpoise occurs for these weights.

Results

Weight (M) of sinker in air	=	gm.
„ (M_0) „ „ „ cold liquid	=	gm.
„ (M_1) „ „ „ hot „	=	gm.
∴ Upthrust of cold liquid ($m_0 = M - M_0$)	=	gm.
and „ „ hot „ ($m_1 = M - M_1$)	=	gm.
Temperature of cold liquid	=	° C.
„ „ hot „	=	° C.
∴ Temperature change (t_c)	=	° C.

$$C_a = \frac{m_0 - m_1}{m_1 \times t_c} = \frac{\quad}{\quad \times \quad}$$

$$= \frac{\quad}{\quad} \text{ per } ^\circ \text{C. for temp. range } \quad ^\circ \text{C. to } \quad ^\circ \text{C.}$$

Theory

Let the volume of the sinker (assumed constant) = V and let the densities of the liquid at the low and high temperatures be respectively D_0 and D_1 . Now by Archimedes' principle the upthrust, or loss in weight, is equal to the weight of the liquid displaced. Hence—

$$\begin{aligned} \text{Upthrust } m_0 \text{ at low temperature} &= VD_0 \\ \text{and } m_1 \text{ „ high „} &= VD_1 \end{aligned}$$

Accordingly
$$\frac{m_0}{D_0} = \frac{m_1}{D_1}$$

But $D_0 = D_1(1 + C_a t)$, where C_a = the coefficient of apparent expansion of the liquid and t_c is the temperature change, and therefore

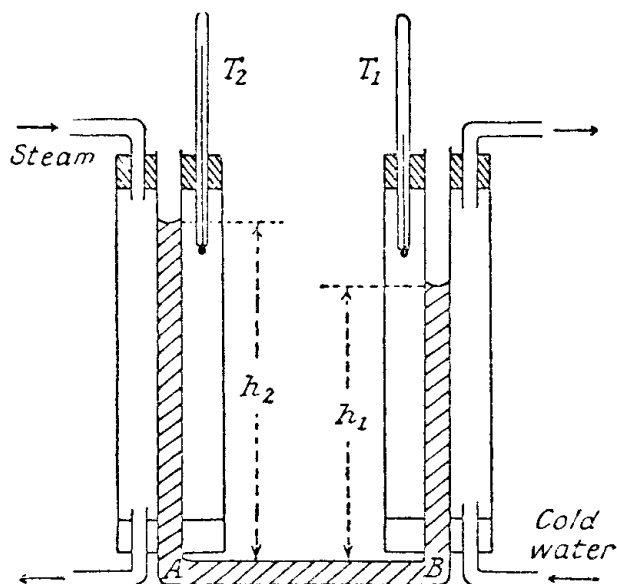
$$\frac{m_0}{m_1} = \frac{D_0}{D_1} = 1 + C_a t$$

giving

$$C_a = \frac{m_0 - m_1}{m_1 \times t_c}$$

EXPERIMENT 55

DETERMINATION OF THE REAL COEFFICIENT OF EXPANSION OF A LIQUID USING A LABORATORY FORM OF DULONG AND PETIT'S APPARATUS



Apparatus

Glass U-tube the limbs of which are surrounded by wide glass tubing bored to carry thermometers and inlet and outlet tubes as shown. Steam from a steam heater is passed through one jacket whilst cold water passes through the other. Also required are : metre rule for measuring the heights of each meniscus above the level of the horizontal portion of the U-tube, steam heater, liquid, e.g. aniline.

Method

Steam from the steam heater is passed through one jacket and cold water from the tap through the other. This is continued until the levels of the liquid in the two limbs of the U-tube are constant. The heights h_1 and h_2 of the cold and hot columns of liquid are then read, and the readings of the two thermometers taken.

Results

Height of cold column (h_1)	=	cm.
„ „ hot „ (h_2)	=	cm.
Temperature of cold water (t_1)	=	° C.
„ „ steam (t_2)	=	° C.
$\therefore C_r = \frac{h_2 - h_1}{h_1 t_2 - h_2 t_1} =$		
		per ° C.

Theory

Let h_1 , h_2 be the heights of the cold and hot columns respectively, and d_1 , d_2 be the densities of the liquid at the low and high temperatures t_1 and t_2 .

Since the columns are hydrostatically balanced, the pressures at the level AB due to each column are the same.

i.e.

$$h_1 d_1 = h_2 d_2$$

or

$$\frac{d_1}{d_2} = \frac{h_2}{h_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now for a given mass of liquid the density varies inversely as the volume,

i.e.

$$\frac{d_1}{d_2} = \frac{v_2}{v_1}$$

but

$$\frac{v_2}{v_1} = \frac{v_0(1 + C_r t_2)}{v_0(1 + C_r t_1)}$$

where v_0 is the volume at 0°C . and C_r is the real coefficient of expansion of the liquid.

Thus

$$\frac{d_1}{d_2} = \frac{1 + C_r t_2}{1 + C_r t_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and from (1) and (2)—

$$\frac{h_2}{h_1} = \frac{1 + C_r t_2}{1 + C_r t_1}$$

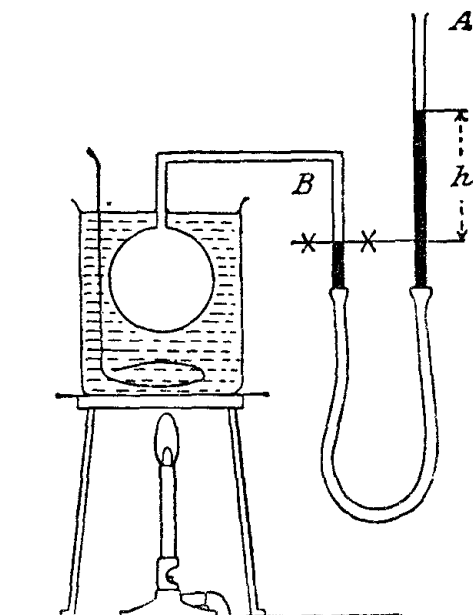
from which

$$C_r = \frac{h_2 - h_1}{h_1 t_2 - h_2 t_1}$$

Note

This method gives the absolute or real coefficient of expansion of the liquid since it is the pressures of the two liquid columns which are balanced against each other. The pressure of a liquid column is independent of the cross-section of the tubing, depending only on the vertical height of the column of liquid.

DETERMINATION OF THE BOILING-POINT OF A LIQUID USING A CONSTANT-VOLUME GAS THERMOMETER



Apparatus

Constant-volume gas thermometer, large beaker, ice, distilled water, liquid, stirrer.

Method

A mark (XX) is made on the scale behind the fixed limb *B* of the manometer, and the bulb is immersed in a mixture of ice and water contained in the beaker. After allowing to stand for several minutes, the movable limb *A* is adjusted until the mercury level in *B* is at the fixed mark. The difference in levels (h_0) between the mercury in the two limbs is then read off. The ice is now replaced with distilled water, which is brought to the boil, and again the limb *A* is adjusted to bring the mercury in *B* to the fixed mark. The new difference in the mercury levels (h_{100}) is read. Having thus determined the two "fixed points" of the thermometer, the difference in the mercury levels (h_t) when the bulb is surrounded by the boiling liquid under investigation is found. Finally the barometric pressure is read from a standard barometer.

Results

Difference in mercury levels :

- | | | | |
|--|---|--|-----|
| (i) with melting ice in beaker (h_0) | = | | cm. |
| (ii) „ boiling water in beaker (h_{100}) | = | | cm. |
| (iii) „ „ liquid „ „ (h_t) | = | | cm. |

$$\therefore t = 100 \frac{h_t - h_0}{h_{100} - h_0} = \underline{\hspace{2cm}} \text{ } ^\circ \text{C.}$$

Theory

The pressure of a given mass of gas heated at constant volume increases according to the following law—

$$P_t = P_0(1 + \alpha t) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where P_0 and P_t are the pressures of the gas at 0°C. and $t^\circ \text{C.}$ respectively, and α is the coefficient of increase of pressure.

If H is the atmospheric pressure and h_0 , h_{100} , and h_t are the difference in mercury levels at 0°C. , 100°C. , and at the temperature of the boiling liquid $t^\circ \text{C.}$, then—

$$P_0 = H + h_0, \quad P_{100} = H + h_{100}, \quad P_t = H + h_t.$$

Hence, since

$$\alpha = \frac{P_{100} - P_0}{P_0 100} = \frac{P_t - P_0}{P_0 t}$$

we have

$$\frac{(H + h_{100}) - (H + h_0)}{(H + h_0)100} = \frac{(H + h_t) - (H + h_0)}{(H + h_0)t}$$

or

$$t = 100 \frac{h_t - h_0}{h_{100} - h_0}.$$

Notes

(1) The boiling-point of the water will not be 100°C. unless the atmospheric pressure is 760 mm. of mercury, and a correction should be made from standard tables for the effect of pressure on the boiling-point.

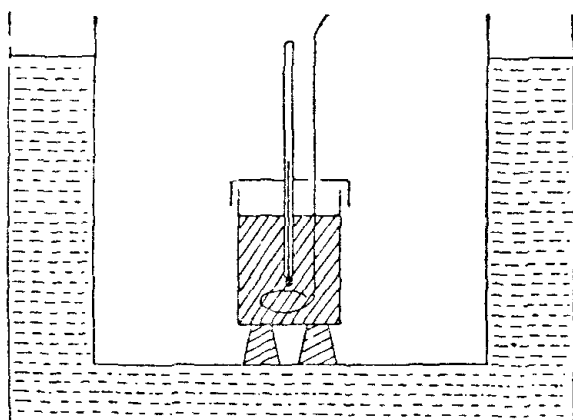
(2) The melting-point of a solid can be obtained using the constant-volume gas thermometer by placing a small quantity of the solid in a thin-walled test-tube alongside the bulb of the thermometer in a water-bath and gradually raising the temperature until the solid melts. The difference in mercury levels is then read off, and the melting-point calculated as above.

(3) From the manometer readings at 0°C. and 100°C. the coefficient of increase of pressure with temperature can be obtained for the gas in the bulb of the thermometer. From the theory above—

$$\begin{aligned} \alpha \text{ (coefficient of increase in pressure)} &= \frac{P_{100} - P_0}{P_0 \times 100} \\ &= \frac{(H + h_{100}) - (H + h_0)}{(H + h_0)100} = \frac{h_{100} - h_0}{(H + h_0)100} \text{ per } ^\circ \text{C.} \end{aligned}$$

(4) Air is usually used in the bulb of the thermometer for experiments such as the one described above. For low-temperature work hydrogen is used, but should be replaced by air or nitrogen at temperatures above 500°C. , as at higher temperatures than this the hydrogen leaks through the bulb.

DETERMINATION OF THE SPECIFIC HEAT OF A LIQUID BY THE METHOD OF COOLING



Apparatus

Copper calorimeter provided with a copper lid (to reduce losses due to convection) and supported on corks inside a double-walled vessel containing cold water between the walls, and sufficiently large to ensure that the temperature of the enclosure does not vary appreciably during the experiment. Also required are: thermometer, stirrer made of stiff copper wire, stopwatch, paraffin, beaker.

Method

The calorimeter is first weighed with the lid and stirrer. It is then filled almost to the top with water that has been heated up to a temperature of about 70° in the beaker. Care must be taken not to heat the liquids to too high a temperature as evaporation will increase the rate of cooling and so cause error. The calorimeter is then supported inside the double-walled jacket, and readings of the temperature are taken every two minutes as the water cools. The accuracy in reading the thermometer will be greatly increased by the aid of a small magnifying-glass. After cooling through a sufficient range of temperature, the calorimeter is removed and weighed with lid, stirrer, and contained water. The calorimeter is now cleaned out and re-filled with the *same volume* of heated paraffin, and data for a cooling curve is obtained as above. The weight of the calorimeter with lid, stirrer, and contained paraffin is also found. Cooling curves for water and paraffin are now drawn, and the gradients taken at corresponding temperatures to give the rates of cooling of the water and paraffin at this temperature from which to calculate the specific heat of paraffin.

Results

t min.	θ° C. water	θ° C. paraffin

Weight of—

calorimeter, lid, and stirrer (m)	=	gm.
-----------------------------------	---	-----

“ “ “ “ + water = gm.

“ “ “ “ + paraffin = gm.

$$\therefore \text{weight of water } (M_1) = \quad \text{gm.}$$

and weight of paraffin (M_2) = gm.

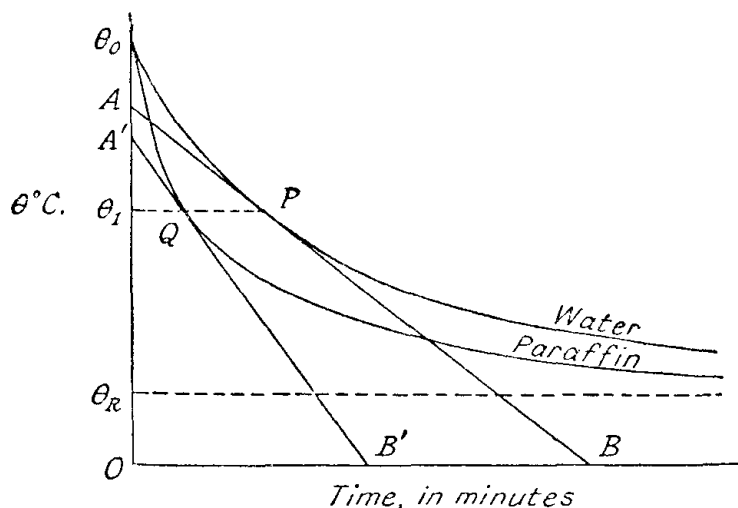
From graph, rates of cooling at $^{\circ}\text{C.}$ are:

$$\left(\frac{d\theta}{dt}\right)_1 = \quad ^\circ \text{C. per min.},$$

and $\left(\frac{d\theta}{dt}\right)_2 =$ ° C. per min.

$$\therefore S =$$

Theory



The rate of loss of heat by a body depends upon the nature and extent of the radiating surface and also upon the temperature of the body and that of its surroundings. The particular form of the law of cooling need not be known if, as in the experiment described here, the conditions of cooling are identical in the two cases. Then, at any given temperature, the rate of loss of heat by the calorimeter and contents must be the same. If M_1 , M_2 be the masses of water (specific heat 1) and paraffin (specific heat S) in the calorimeter, and $\left(\frac{d\theta}{dt}\right)_1$, $\left(\frac{d\theta}{dt}\right)_2$ be the corresponding rates of fall of temperature at some temperature θ_1 (say), then if m be the mass of the copper calorimeter, lid, and stirrer of specific heat 0.1, it follows that—

$$(M_1 + m \times 0.1)\left(\frac{d\theta}{dt}\right)_1 = (M_2 S + m \times 0.1)\left(\frac{d\theta}{dt}\right)_2 \quad \left[\text{where } \left(\frac{d\theta}{dt}\right)_1 = \frac{OA}{OB} \text{ and } \left(\frac{d\theta}{dt}\right)_2 = \frac{OA'}{OB'} \right]$$

from which S can be calculated.

Note

A particular form of cooling law is that given by Newton, which states that the rate of loss of heat by a body is directly proportional to the excess temperature of the body above that of its surroundings. This law is true for quite large temperature differences *provided* the body cools under the influence of a strong draught—otherwise it is true for only small temperature excesses. Newton's law may be verified by obtaining a cooling curve for a calorimeter containing water placed near an open window. By taking gradients at 5° intervals on this curve, and plotting the values of $\left(\frac{d\theta}{dt}\right)$ as ordinates against values of the excess temperature $(\theta - \theta_r)$ as abscissae, a straight-line graph passing through the origin should be obtained if Newton's law is correct.

Newton's law may be written $-\frac{d\theta}{dt} = k\theta$ (where θ represents the excess temperature).

Rearranging,

$$\frac{d\theta}{\theta} = -k dt.$$

Integrating,

$$\log_e \theta = -kt + \text{const.}$$

Now

$$\theta = \theta_0 \text{ (initial temp.) when } t = 0, \therefore \text{the const.} = \log_e \theta_0.$$

Hence

$$\log_e \theta - \log_e \theta_0 = -kt$$

$$\text{or } \theta = \theta_0 e^{-kt}.$$

This is an example of logarithmic variation. Another example is given in Expt. 95.

EXPERIMENT 58

DETERMINATION OF THE SPECIFIC HEAT OF A BAD CONDUCTOR

Apparatus

Copper calorimeter with stirrer (of thick copper wire), double-walled enclosure with cold water between the walls, thermometer reading in $\frac{1}{10}$ ths $^{\circ}\text{C}$., stop-watch, steam heater, piece of rubber, e.g. large rubber stopper.

Method

The rubber stopper is weighed and placed in the steam heater where it is left for a period of at least 30 minutes until its temperature reaches that of the steam. During this time the calorimeter (with stirrer) is weighed empty, and then about half full with water. The calorimeter and contents are placed on corks inside the constant-temperature enclosure, and the temperature of the water taken. The hot rubber stopper is now quickly transferred to the waiting calorimeter, and the stop-watch started. Readings of the temperature are taken at minute intervals, and are continued after the mixture has attained its highest temperature until the calorimeter and contents cool through about 5°C . A temperature-time graph is now drawn which is corrected for radiation losses in the manner indicated so as to obtain the corrected final temperature of the mixture for use in the heat equation for calculating the specific heat.

Results

Time in min.	$\theta^{\circ} \text{C.}$	Cooling in ° C. per min. (from Fig. 2)	Correction	Corrected temp.

Weight of rubber stopper (m) = gm.

Weight of calorimeter and stirrer (w) = gm.

" " " " " and water = gm.

∴ „ water (M) = gm.

Temperature of steam (T'') = °C.

„ „ cold water (t) = °C.

,, mixture (corrected) (T) = °C.

Heat lost by rubber stopper (of specific heat S) = Heat gained by calorimeter and contents,
i.e. $mS(T' - T) = (M + w \times 0.1)(T - t)$

$$mS(T' - T) = (M + w \times 0.1)(T - t)$$

$$\therefore S =$$

Theory

For accurate calorimetry, a correction for radiation losses should be made in fixing the temperature of the mixture. In dealing with metals where the transfer of heat takes place quickly, the correction is usually ignored. In the case of a badly conducting solid like rubber or glass, however, a considerable time elapses before the hot solid gives up its

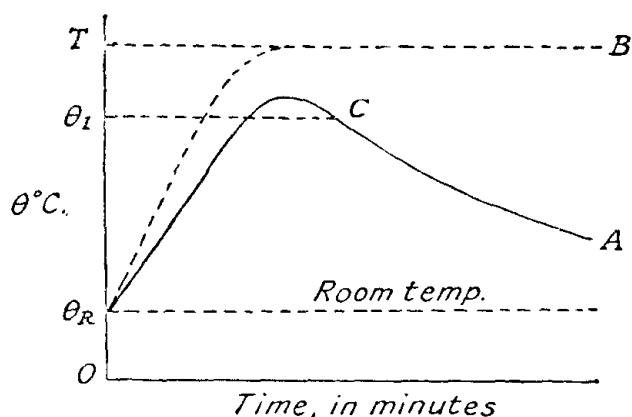


FIG. 1.

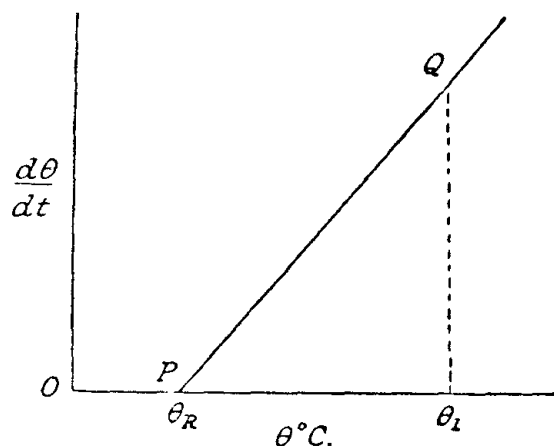


FIG. 2.

heat to the water. During this time there is an appreciable loss of heat by radiation from the calorimeter, and the final temperature indicated is well below the value that should be used in calculating the result.

To obtain the corrected temperature, the following procedure is adopted:

From the observed temperature-time curve (graph A in Fig. 1), the rate of cooling at some point C, corresponding to a temperature θ_1 , is obtained by drawing a tangent to the curve at C. Since the rate of cooling at the room temperature (θ_R) is zero, a straight-line graph (Fig. 2) can be drawn showing rate of cooling in $^\circ\text{C. per min.}$ against $\theta^\circ\text{C.}$ from these two points. By Newton's law of cooling, the rate of cooling at any other temperature is proportional to the excess temperature of the body over the room temperature, and hence from this straight line the rate of cooling for any temperature can be read. From Fig. 1 the mean temperature during the first minute interval is taken, and the rate of cooling in $^\circ\text{C. per min.}$ corresponding to this temperature is obtained from Fig. 2. This will give the amount by which the ordinate at the end of the minute interval in graph A must be increased to obtain the corrected temperature. The correction for the next minute interval is obtained in the same way, and the ordinate at the end of this interval is increased by the sum of the corrections for the two intervals. Proceeding in this way the corrected curve B is obtained from which the corrected final temperature T' is read.

EXPERIMENT 59

DETERMINATION OF THE MELTING-POINT OF A SOLID BY THE COOLING-CURVE METHOD

Apparatus

Small copper calorimeter with copper lid with hole for thermometer, double-walled enclosure with cold water between the walls, supporting corks, thermometer 0–100° in $\frac{1}{10}$ ths °C., pure naphthalene, beaker, stop-watch.

Method

The small copper calorimeter is about half filled with solid naphthalene which is melted by placing the calorimeter in a beaker containing boiling water. When all the naphthalene has melted, the calorimeter is removed, dried on the outside, and supported inside the constant-temperature enclosure as in Expt. 57. The copper lid is placed on the calorimeter and the thermometer inserted into the liquid naphthalene through a hole bored in the lid. Data for a cooling curve is now obtained, the temperature of the naphthalene being taken at half-minute intervals, and the readings continued for some time after solidification has been completed. A graph is then drawn of temperature against time, and the melting-point read from the horizontal portion of the graph.

Results

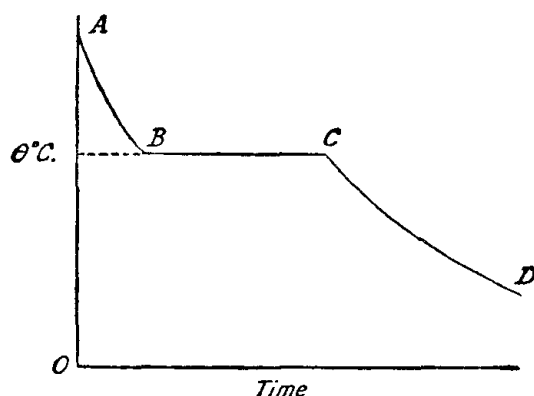
$\theta^{\circ} \text{C.}$	Time	$\theta^{\circ} \text{C.}$	Time

From the graph :

Melting-point = °C.

Theory

The cooling curve obtained under the conditions of the experiment is as indicated in the diagram. In the stage AB the substance is liquid, BC is the solidifying stage, and CD represents the cooling of the solid substance. As the calorimeter is cooling in a constant-temperature enclosure, the sections AB and CD of the curve are determined to a close approximation by Newton's law of cooling. During the process of solidification, however



the temperature is maintained constant as the latent heat released during this process offsets the loss of heat due to radiation from the surface of the calorimeter. The temperature at which this occurs is the melting-point required.

Note

A value of the latent heat of fusion of the substance can be obtained by finding the rate of cooling at the commencement and finish of the solidifying stage. Theoretically these values are the same, since the temperature is the same at both points, but it will be found difficult to obtain a gradient exactly at B or at C . If, however, the rate of cooling at a point near B is $\left(\frac{d\theta}{dt}\right)_1$ and that at a point near C is $\left(\frac{d\theta}{dt}\right)_2$, and S_1 and S_2 are the specific heats in the liquid and solid states respectively, and if a mass M of the substance is contained in the calorimeter whose mass (with lid) is m , then the average rate of cooling at the freezing-point is—

$$\frac{1}{2} \left\{ (MS_1 + m \times 0.1) \left(\frac{d\theta}{dt}\right)_1 + (MS_2 + m \times 0.1) \left(\frac{d\theta}{dt}\right)_2 \right\} . \quad . \quad . \quad (1)$$

and if the stage BC is completed in a time t seconds, the total loss of heat during solidification

$$= \frac{t}{2} \left\{ (MS_1 + m \times 0.1) \left(\frac{d\theta}{dt}\right)_1 + (MS_2 + m \times 0.1) \left(\frac{d\theta}{dt}\right)_2 \right\} . \quad . \quad . \quad (2)$$

and this must equal the total latent heat evolved, that is

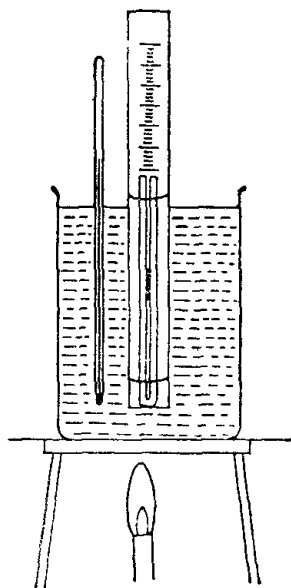
$$ML . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence from (2) and (3) we have—

$$L = \frac{t}{2} \left\{ (S_1 + m \times 0.1) \left(\frac{d\theta}{dt}\right)_1 + (S_2 + m \times 0.1) \left(\frac{d\theta}{dt}\right)_2 \right\} \text{ cal./gm.}$$

EXPERIMENT 60

DETERMINATION OF THE SATURATED VAPOUR PRESSURE OF WATER AT DIFFERENT TEMPERATURES



Apparatus

Length of thick-walled capillary tubing closed at one end and attached to a millimetre scale (e.g. a ruler) by rubber bands. Deep beaker, thermometer.

Method

The capillary tubing is first gently warmed over the bunsen flame, and the open end is then immersed in some cold water. A thread of water about 2 cm. long is drawn in the tube, and should occupy a position near the middle at room temperature. The capillary tubing is now attached to the millimetre scale and placed in a deep beaker containing water so that the open end of the tube is open to the air. The water-bath is slowly heated, and the length (l) of the moist air in the tube (measured from the closed end to the bottom end of the water thread) is read at different temperatures. The bunsen is then turned off, and a set of readings of the length of the moist air column is taken at corresponding temperatures as the water cools. The atmospheric pressure (P) is taken from a standard barometer.

Results

Barometric reading (P) = mm. of mercury.

$t^{\circ}\text{C.}$	Readings of l		Mean l	S.V.P.
	Temp. rising	Temp. falling		

Theory

The quantity of heat conducted across a section of a substance is proportional to the area of the section, the temperature gradient at the section, and the time considered.

Thus
$$H = kA \frac{d\theta}{dx} \cdot t \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where k is the amount of heat crossing unit area of cross section of the substance per second for unit temperature gradient, and is known as the *coefficient of thermal conductivity*.

In Searle's method it is assumed that all the heat that enters one end of the bar is conducted along the bar to be extracted at the other end, thus giving rise to a uniform temperature gradient. Then if θ_1 , θ_2 , θ_3 , and θ_4 are the readings of the thermometers T_1 , T_2 , T_3 , and T_4 , in the "steady state," and if M gm. of water emerge from the copper spiral in t seconds, then equation (1) becomes—

$$M(\theta_3 - \theta_4) = k \cdot \frac{\pi D^2}{4} \cdot \left(\frac{\theta_1 - \theta_2}{d} \right) t$$

where D is the diameter of the copper bar, and d is the distance between the two thermometers T_1 and T_2 ,

i.e.
$$k = \frac{4Md(\theta_3 - \theta_4)}{\pi D^2(\theta_1 - \theta_2)t}.$$

Results

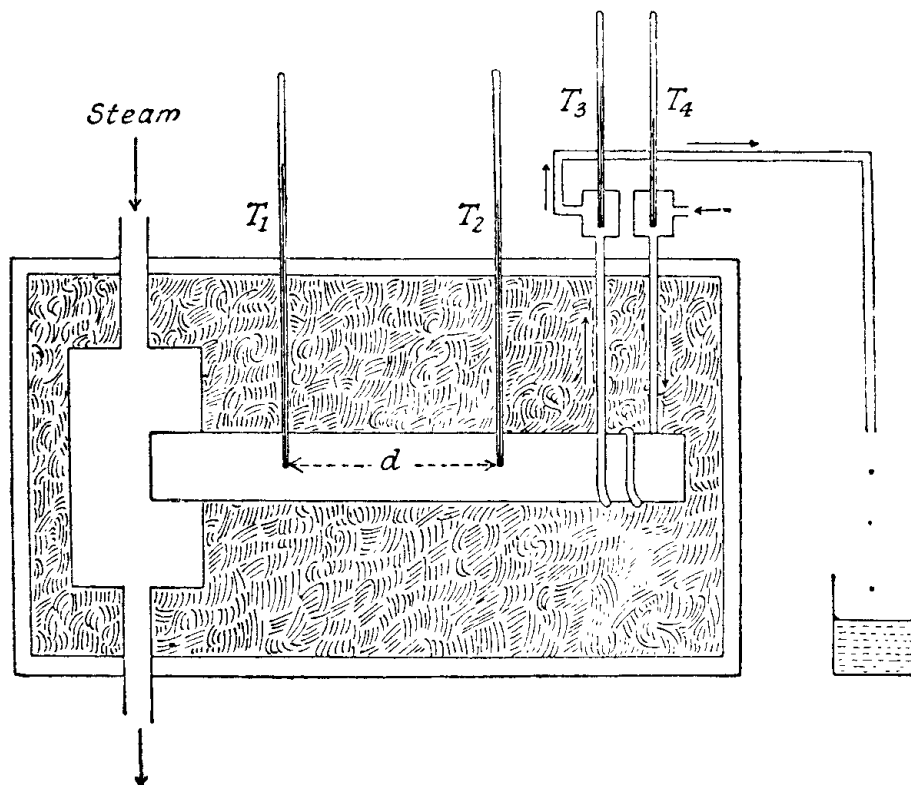
Mass of water collected =	gm.			
Time of flow =	sec.			
Distance (d) =	cm.			
Diameter of bar =	cm.			
Readings of thermometers at steady state:	T_1	T_2	T_3	T_4
Before interchange				
After interchange				
Mean temperatures				
$\therefore k =$ _____	cal./sq. cm./unit temperature gradient/sec.			

Note

Electrically heated models of Searle's apparatus are available by which it is possible to take observations at the hot end of the bar. Thus if I amp. are supplied to the heater under a P.D. of V volts, the heat generated in t sec. is $\frac{IVt}{4.2}$ cal. This amount of heat should equal the amount extracted in t seconds at the cool end of the bar at the steady state if there is no loss of heat from the sides of the bar. As there is no perfect heat insulator, such a loss will occur in some measure in the present case. Thus readings from the hot end of the bar will give a value of k which is too large whilst readings taken at the cool end will give k too small. Consequently a mean of the two results obtained with the electrically heated metal bar will give a better result for k than the one obtained by the method indicated above.

EXPERIMENT 61

DETERMINATION OF THE THERMAL CONDUCTIVITY OF COPPER BY SEARLE'S METHOD



Apparatus

Standard form of Searle's apparatus for use with steam heater. The apparatus consists of a stout copper bar fitted into a steam-chest at one end, and having a narrow copper tube coiled round the other end through which cold water from a constant-head device passes. The temperature of this water as it enters and leaves is taken by two $\frac{1}{10}^{\circ}\text{C}$. thermometers T_4 and T_3 . The whole apparatus is lagged with felt and contained in a wooden box. Two holes are drilled in the bar a distance d cm. apart in which are placed the $\frac{1}{10}^{\circ}\text{C}$. thermometers T_1 and T_2 dipping in mercury to ensure good contact. Also required are beaker, stop-clock and callipers.

Method

Steam from the steam-heater is passed through the steam-chest, and the constant-head device is adjusted to give only a small rate of flow through the coiled tube. The rate of flow is obtained by weighing the amount of water collected from the coil in a measured time. When all the temperatures have become steady (usually after 30 minutes or more), the readings of the four thermometers are taken. Thermometers T_1 and T_2 are now interchanged, as also are T_3 and T_4 , and the temperature readings again taken. Finally the diameter of the bar is found by callipers, and the distance between the two thermometers T_1 and T_2 measured.

Theory

The quantity of heat conducted across a section of a substance is proportional to the area of the section, the temperature gradient at the section, and the time considered.

Thus
$$H = kA \frac{d\theta}{dx} \cdot t \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where k is the amount of heat crossing unit area of cross section of the substance per second for unit temperature gradient, and is known as the *coefficient of thermal conductivity*.

In Searle's method it is assumed that all the heat that enters one end of the bar is conducted along the bar to be extracted at the other end, thus giving rise to a uniform temperature gradient. Then if θ_1 , θ_2 , θ_3 , and θ_4 are the readings of the thermometers T_1 , T_2 , T_3 , and T_4 , in the "steady state," and if M gm. of water emerge from the copper spiral in t seconds, then equation (1) becomes—

$$M(\theta_3 - \theta_4) = k \cdot \frac{\pi D^2}{4} \cdot \left(\frac{\theta_1 - \theta_2}{d} \right) t$$

where D is the diameter of the copper bar, and d is the distance between the two thermometers T_1 and T_2 ,

i.e.
$$k = \frac{4Md(\theta_3 - \theta_4)}{\pi D^2(\theta_1 - \theta_2)t}.$$

Results

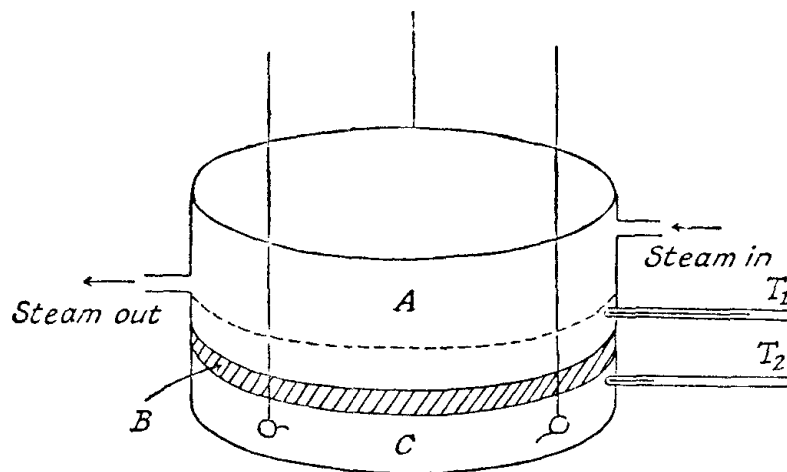
Mass of water collected =	gm.			
Time of flow =	sec.			
Distance (d) =	cm.			
Diameter of bar =	cm.			
Readings of thermometers at steady state:	T_1	T_2	T_3	T_4
Before interchange				
After interchange				
Mean temperatures				
$\therefore k =$ _____	cal./sq. cm./unit temperature gradient/sec.			

Note

Electrically heated models of Searle's apparatus are available by which it is possible to take observations at the hot end of the bar. Thus if I amp. are supplied to the heater under a P.D. of V volts, the heat generated in t sec. is $\frac{IVt}{4.2}$ cal. This amount of heat should equal the amount extracted in t seconds at the cool end of the bar at the steady state if there is no loss of heat from the sides of the bar. As there is no perfect heat insulator, such a loss will occur in some measure in the present case. Thus readings from the hot end of the bar will give a value of k which is too large whilst readings taken at the cool end will give k too small. Consequently a mean of the two results obtained with the electrically heated metal bar will give a better result for k than the one obtained by the method indicated above.

EXPERIMENT 62

DETERMINATION OF THE COEFFICIENT OF THERMAL CONDUCTIVITY OF EBONITE BY LEES' DISC METHOD



Apparatus

Standard laboratory form of Lees' Disc apparatus. This consists of a cylindrical slab of metal *C* (of copper or brass) suspended by strings from a heavy stand. On this rests a hollow cylinder *A* through which steam from a steam heater is passed, and the specimen in the form of a thin disc *B* of the same diameter is placed between *A* and *C*. A hole is bored near the bottom of cylinder *A* and another in *C* to take two $\frac{1}{10}^{\circ}\text{C}$. thermometers, and the two metal cylinders are nickel plated to give them the same emissive power. A stop-watch and a screw gauge are also required.

Results

Readings of thermometers in steady state : T_1

T_2

Data for cooling curve :

Before interchanging

After interchanging

Mean temperatures

Diameter of ebonite disc = cm.

Thickness „ „ „ = cm.

Mass of metal slab *C* = gm.

Specific heat of metal *C* (from tables) =

Rate of cooling of slab *C* at $\theta_2^{\circ}\text{C}$.
(from graph) = $^{\circ}\text{C}$. per sec.

Temp. $^{\circ}\text{C}$.	Time in sec.

$\therefore k =$ _____ cal./sq. cm./unit temp. gradient/sec.

Method

The apparatus is suspended as shown in the diagram and with the flat surfaces of the discs horizontal. Steam is passed through the cylinder A , and the temperatures indicated by the two thermometers T_1 and T_2 are read when the steady state has been reached (30 to 60 minutes). T_1 and T_2 are then interchanged, and their temperatures again read when steady. Cylinder A is now removed and a bunsen flame is allowed to play on the bottom surface of the slab C until T_2 records a temperature up to 10° C. higher than that recorded in the steady state. It is now allowed to cool, and readings of the temperature are taken at half-minute intervals until the temperature falls to about 10° C. below the steady state temperature. The diameter of the ebonite disc is now measured, and its thickness found by screw gauge. Finally the mass of the metal slab C is determined.

Theory

When the steady state has been attained, the rate at which heat is conducted across the ebonite disc is equal to the rate at which it is emitted from the exposed surfaces of the metal slab C . If k is the thermal conductivity of the ebonite disc, D its diameter and d its thickness, and the readings of the thermometers T_1 and T_2 in the steady state are θ_1 and θ_2° C., then the rate at which heat is conducted across the disc of ebonite is—

$$k \cdot \frac{\pi D^2}{4} \cdot \left(\frac{\theta_1 - \theta_2}{d} \right) \text{ calories per second} \quad . \quad . \quad . \quad . \quad (1)$$

If the mass of the metal slab C is M , its specific heat S , and the rate of cooling at θ_2° C. is $\left(\frac{d\theta}{dt} \right)_2$ (obtained by drawing a tangent to the cooling curve at θ_2), then the rate of loss of heat from the lower face and the sides of the slab C is—

$$MS \left(\frac{d\theta}{dt} \right)_2 \text{ calories per sec.} \quad . \quad . \quad . \quad . \quad (2)$$

Hence equating (1) and (2) we have

$$k \cdot \frac{\pi D^2}{4} \cdot \left(\frac{\theta_1 - \theta_2}{d} \right) = MS \left(\frac{d\theta}{dt} \right)_2 \quad . \quad . \quad . \quad . \quad (3)$$

from which k may be determined.

Note

Since by Newton's law of cooling the rate of loss of heat is proportional to the excess temperature of a body over that of its surroundings, equation (3) above may be written—

$$k \cdot \frac{\pi D^2}{4} \cdot \left(\frac{\theta_1 - \theta_2}{d} \right) = \text{const.} (\theta_2 - \theta_0)$$

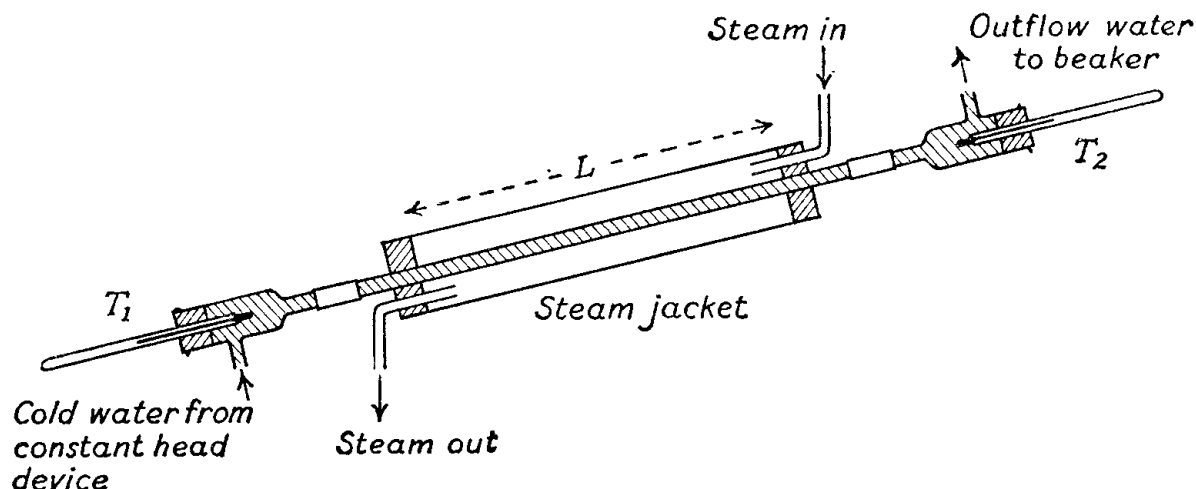
where θ_0 is the air temperature.

Hence if for a specimen of another material of thermal conductivity k' and thickness d' , the steady state temperatures are θ_1' and θ_2' , a comparison of the thermal conductivities of the two specimens can be obtained without the necessity of proceeding with the second part (cooling curve) of the experiment.

Thus

$$\frac{k}{k'} = \frac{d(\theta_2 - \theta_0)(\theta_1' - \theta_2')}{d'(\theta_2' - \theta_0)(\theta_1 - \theta_2)}$$

DETERMINATION OF THE THERMAL CONDUCTIVITY OF GLASS



Apparatus

Length of glass tubing about 50 cm. long and of internal diameter 1 cm. surrounded by a jacket through which steam is passed from a steam heater. A slow stream of cold water from a constant-head device (see Expt. 26) passes along the tube which is tilted slightly to eliminate air-pockets. Two glass connecting pieces are attached at the ends of the tube to carry $\frac{1}{10}^{\circ}\text{C}$. thermometers; these connecting pieces may be made from test-tubes with short lengths of narrow glass tubing fixed at the bottoms and sides as indicated. Also required are: a beaker to collect the outflow water, a stop-clock, and a travelling microscope.

Method

Steam is passed through the steam-jacket, and the rate of flow of the water from the constant-head device is adjusted to give an appreciable difference of temperature between the thermometers T_1 and T_2 in the steady state. The rate of flow is obtained by weighing the quantity of water (M) emerging in a measured time (t). The temperatures of T_1 and T_2 are measured in the steady state, and further readings of the inflow and outflow temperatures are taken with the same rate of flow but with the two thermometers interchanged. The length (L) of the tube inside the steam-jacket is measured, and the internal and external radii obtained by a travelling microscope.

Results

$L =$ cm. $r_1 =$ cm. $r_2 =$ cm.

$M =$ gm. $t =$ sec.

Temperature of steam (θ') = $^{\circ}\text{C}$. (from tables)

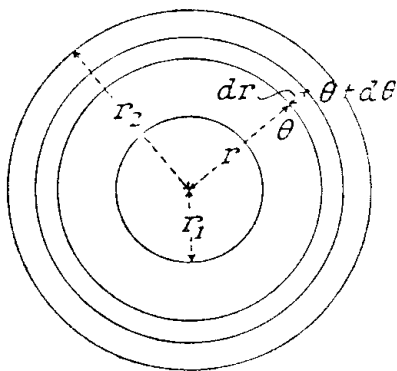
Readings of thermometers at steady state: T_1 T_2

Before interchange

After interchange

Mean readings

$\therefore k =$ _____ cal./sq. cm./unit temperature gradient/sec.



Theory

Consider the heat flow across a thin section of length L and bounded by surfaces of radii r and $r + dr$. If the temperatures at these surfaces are θ and $(\theta + d\theta)$ respectively, the inward flow of heat in t sec. is—

$$H = k \cdot 2\pi r L \cdot \frac{d\theta}{dr} \cdot t$$

Re-arranging and integrating between the limits r_1 and r_2 (internal and external radii) for r , and θ' and θ_0 (steam temperature and inside temperature respectively) for θ we have—

$$H \int_{r_1}^{r_2} \frac{dr}{r} = 2\pi k L t \int_{\theta_0}^{\theta'} d\theta$$

i.e.

$$H \log_e \frac{r_2}{r_1} = 2\pi k L t (\theta' - \theta_0)$$

or

$$k = \frac{H}{2\pi L t (\theta' - \theta_0)} \cdot \log_e \frac{r_2}{r_1}$$

Now if M gm. of water enter at a temperature of θ_1 and emerge at θ_2 in t sec., then $H = M(\theta_2 - \theta_1)$ and $\theta_0 = \frac{\theta_1 + \theta_2}{2}$,

then

$$k = \frac{M(\theta_2 - \theta_1)}{2\pi L t \left\{ \theta' - \left(\frac{\theta_1 + \theta_2}{2} \right) \right\}} \cdot \log_e \frac{r_2}{r_1}$$

Notes

(1) The above theory also applies to the determination of the thermal conductivity of rubber in the form of rubber tubing. A length L cm. is coiled in a large copper calorimeter of mass m gm. containing M gm. of water at a temperature θ_1 (conveniently cooled below room temperature to reduce radiation losses), and heated by passing steam (at a temperature of θ') through the tube until the temperature of the water rises to θ_2 in a time t sec. k is calculated as above except that in this case, $H = (M + m \times 0.1)(\theta_2 - \theta_1)$.

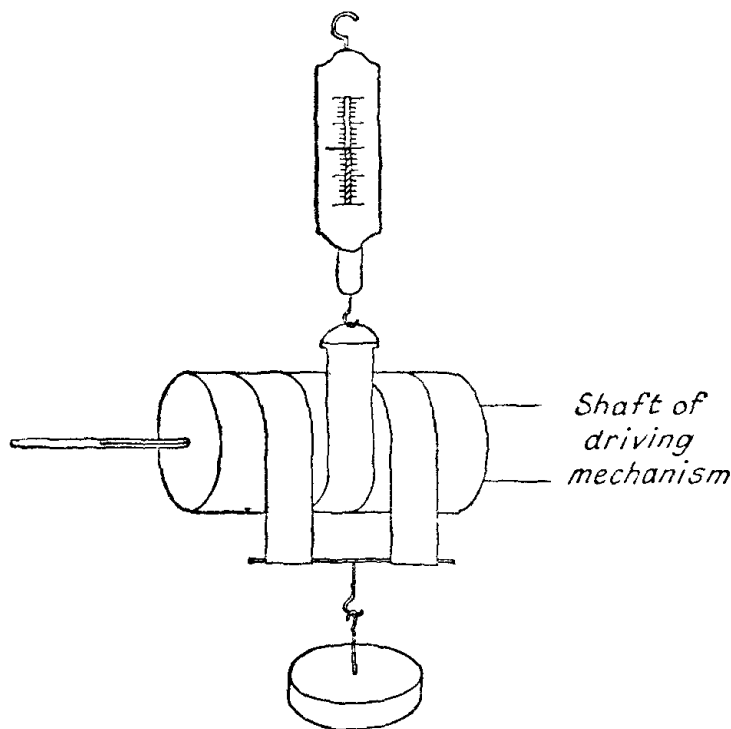
(2) In the case of thin-walled tubing the value of k may be found by considering the heat conducted across a section of mean radius $\frac{r_1 + r_2}{2}$ under a constant-temperature gradient. Then

$$H = k \cdot 2\pi \left(\frac{r_1 + r_2}{2} \right) L \cdot \frac{\left\{ \theta' - \left(\frac{\theta_1 + \theta_2}{2} \right) \right\}}{r_2 - r_1}$$

which gives

$$k = \frac{H(r_2 - r_1)}{\pi(r_1 + r_2)Lt \left\{ \theta' - \left(\frac{\theta_1 + \theta_2}{2} \right) \right\}}$$

DETERMINATION OF THE MECHANICAL EQUIVALENT OF HEAT BY CALLENDAR'S METHOD



Apparatus

Laboratory form of Callendar's apparatus supplied with solid copper calorimeter of known mass. A silk friction band encircles the copper calorimeter, its upper end being attached to a spring balance, and its lower end supporting a large weight. The calorimeter is rotated either manually or by electric motor, and the number of revolutions is counted by a revolution indicator attached to the apparatus. A hole is bored in the calorimeter into which fits a $\frac{1}{10}^{\circ}\text{C}$. thermometer whose bulb is dipped in oil to ensure good contact. Also required are stop-watch and callipers.

Method

The position of the spring balance is adjusted until the upward tension is just sufficient to "float" the attached load when the calorimeter is rotated at a steady rate. The rise in temperature is carefully observed for a given number of revolutions, and the time taken for this is noted. The rotation of the calorimeter is continued at the end of the experiment until the temperature rises by a further 1°C . The calorimeter is now allowed to cool, and the time is taken for the temperature to fall to 1°C . below the final temperature of the experiment. Finally the radius of the calorimeter is taken with callipers.

Note.—If the upward tension varies during the experiment, readings of the spring balance should be taken every 50 revolutions (say), and the mean value taken.

Results

Mass of calorimeter	=	gm.
Specific heat of copper	=	
Initial temperature	=	$^{\circ}\text{C}$.
Final temperature	=	$^{\circ}\text{C}$.
Number of revolutions	=	
Radius of calorimeter	=	cm.
Time of experiment	=	min.
Rate of cooling at end of experiment	=	$^{\circ}\text{C}$. per min.
$\therefore J =$ _____ ergs/calorie.		

Theory

When work is done on a body under conditions such that it is completely converted into heat, the number of heat units (H) developed bears a constant ratio to the amount of work (W) done. This ratio is known as the mechanical equivalent of heat (J).

Thus

$$J = \frac{W}{H}$$

In the experiment in question, work done against the friction of the silk brake-band is converted into heat which raises the temperature of the copper calorimeter. If T_1 is the upward tension, and T_2 the downward tension in gm.-wt., the force of friction $= (T_2 - T_1)g$ dynes, and if r is the radius of the copper cylinder, the frictional couple $= (T_2 - T_1)gr$ dyne-cm. Hence for n revolutions the work done

$$= (T_2 - T_1)gr2\pi n \text{ ergs.}$$

If θ_1 is the initial temperature of the copper calorimeter (of mass M and specific heat S), and θ_2 is the final temperature, the heat developed

$$= MS(\theta_2 - \theta_1 + \delta\theta) \text{ calories,}$$

where $\delta\theta$ is the correction for radiation losses during the experiment (see note below).

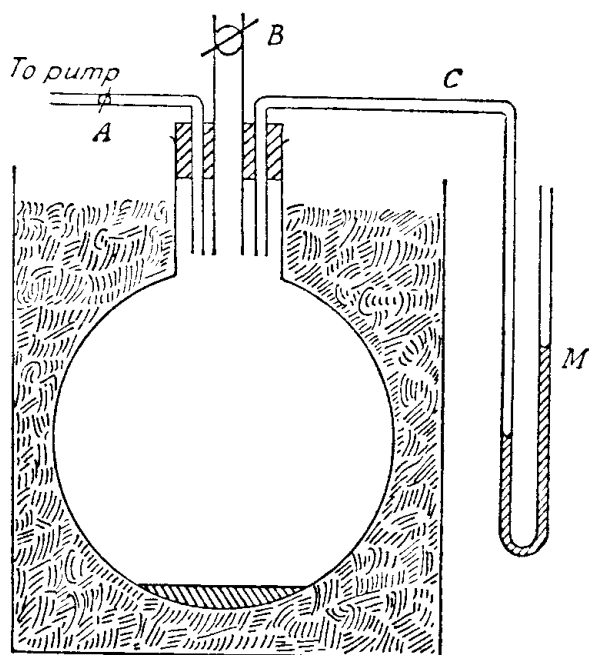
Hence

$$J = \frac{(T_2 - T_1)gr \cdot 2\pi n}{MS(\theta_2 - \theta_1 + \delta\theta)} \text{ ergs/cal.}$$

Note

Since the rise in temperature of the calorimeter is virtually constant during the experiment, the mean rate of loss of heat is half the rate of loss at the final temperature. The rate of cooling at the final temperature (θ_2) is obtained as indicated in the experiment, let it be represented by $x^\circ \text{ C. per min.}$ Then for the t min. of the experiment the cooling correction ($\delta\theta$) is $t \frac{x^\circ}{2} \text{ C.}$

DETERMINATION OF THE RATIO OF THE SPECIFIC HEATS OF A GAS BY CLÉMENT AND DESORMES' METHOD



Apparatus

The apparatus consists of a large vessel (of volume 10 litres or more) containing a little concentrated sulphuric acid to keep the gas inside dry. The mouth of the vessel is closed by a rubber bung through which passes a narrow tube *A* connected to a pump, a wide tube *B* fitted with a stop-cock, and the connecting tube *C* to an oil or sulphuric-acid manometer (*M*). The vessel is placed in a box and lightly packed with cotton-wool to exclude draughts.

Method

Some dry air is pumped into the vessel, and when it has taken up the temperature of its surroundings, the manometer reading is taken (a pressure increase of about 30 cm. of oil is convenient). The stop-cock *B* is now opened, and closed again after about 1 second. After the adiabatically cooled gas has once again acquired the temperature of its surroundings, the manometer is again read. Finally the atmospheric pressure is taken from a standard barometer. The experiment is repeated with different values of the initial pressure of the gas inside the vessel.

Results

Atmospheric pressure (*P*) = cm. of mercury.

in cm. of mercury p_1	in cm. of mercury p_2	γ
Average =		

Theory

The principle of the method is to produce an adiabatic cooling by sudden expansion of a gas, and then to follow this by an increase of pressure at constant volume until the gas attains its original temperature.

Let the gas that remains in the vessel initially occupy a volume v which is rather less than the volume V of the vessel. If then p_1 and p_2 are respectively the initial and final pressures of the gas, and P is the atmospheric pressure, we have for the adiabatic change—

$$p_1 v^\gamma = P V^\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and for the isothermal change

$$p_1 v = p_2 V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2)

$$\frac{p_1}{P} = \left(\frac{p_1}{p_2} \right)^\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

or taking logs

$$\log p_1 - \log P = \gamma (\log p_1 - \log p_2)$$

i.e.

$$\gamma = \frac{\log p_1 - \log P}{\log p_1 - \log p_2}.$$

Notes

(1) It is important to see that the tube B is wide enough to ensure quick expansion and thus obtain true adiabatic cooling, and further to avoid oscillations which occur with narrow tubes.

(2) A knowledge of γ gives information regarding the atomicity of the gas—e.g. for a monatomic gas $\gamma = 1.66$, for a diatomic gas $\gamma = 1.4$, and for a triatomic gas $\gamma = 1.3$, etc.

(3) For small pressure differences the expression for γ can be given in a simpler form as follows:

Let $p_1 = P + h_1$ and $p_2 = P + h_2$ where h_1 and h_2 are small compared with P . Equation (3) above then becomes

$$\frac{P + h_1}{P} = \left(\frac{P + h_1}{P + h_2} \right)^\gamma$$

that is

$$1 + \frac{h_1}{P} = \left(\frac{1 + \frac{h_1}{P}}{1 + \frac{h_2}{P}} \right)^\gamma$$

or

$$\left(1 + \frac{h_1}{P} \right)^{\gamma-1} = \left(1 + \frac{h_2}{P} \right)^\gamma$$

Expanding by the binomial theorem and neglecting the squares and higher powers of $\frac{h_1}{P}$ and $\frac{h_2}{P}$ we have,

$$1 + (\gamma - 1) \frac{h_1}{P} + \dots = 1 + \gamma \frac{h_2}{P} + \dots$$

from which

$$\gamma = \frac{h_1}{h_1 - h_2}.$$