# A LABORATORY MANUAL OF PHYSICS

FOR ADVANCED LEVEL CERTIFICATE,
SCHOLARSHIP AND INTERMEDIATE SCIENCE STUDENTS

BY

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## Mechanics and Properties of Matter
## Part 1

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# INTRODUCTION

## 1. General

To a large extent, experimental Physics is concerned with the quantitative determination of physical constants, e.g. the specific heat of a solid, acceleration of gravity, coefficient of thermal conductivity, etc. Some experiments, however, are designed to verify known laws, others to obtain the empirical relationships between two or more quantities. In every case, accurate and methodical observations are necessary, and these should be taken with an intelligent realization of the capabilities of the apparatus provided. A course in practical Physics is designed to give the student the opportunity of acquiring the necessary skill and technique in the manipulation of apparatus, and the use and understanding of the instruments employed.

## 2. Records

A faithful record of all observations taken should be made in a book specially reserved for the purpose. This book should be at hand whilst carrying out the experiment, and full details of the results should be entered with relevant remarks where necessary. A book should also be available in which a full record of the experiment is kept, with diagrams and full practical details, in a manner similar to that followed in the pages of this book. If the book is not ruled for graphical work, a sheet of graph paper can be interleaved when necessary between the double page of the record.

In stating the final result of the experiment, it is important to remember that the statement is incomplete if unaccompanied by the units in terms of which the quantity is measured. Where there is no special name for the unit, it is usual to give the dimensions of the quantity in reference to the three fundamental units of mass, length, and time. In the C.G.S. system of units, these are the gramme, centimetre, and second respectively. Thus acceleration expressed in this system would be written as cm., sec.$^{-2}$ or centimetres per second per second; similarly, density would be written as gm., cm.$^{-3}$ or grammes per cubic centimetre.

If the quantity measured is very large or very small, it is usual to give the measure with one significant figure before the decimal point, multiplied by a power of 10. Thus Young's Modulus for Aluminium would be given as $7 \cdot 0 \times 10^{11}$ dynes per sq. cm. and the wavelength of the $D_1$ line of sodium as $5 \cdot 896 \times 10^{-5}$ cm.

## 3. Errors and Degree of Accuracy

All measurements taken during an experiment are subject to the errors of observation and also to the errors inherent in the apparatus. Errors of observation can be minimized by obtaining several readings carefully and methodically, and then taking their arithmetical mean. In the case of instrumental errors, it must be realized that not every piece of apparatus is capable of giving measurements to a high degree of accuracy. The capabilities of each component used should be considered, and the degree of accuracy of the final result will not be greater than that of the least reliable instrument used. In determining the specific heat of a solid by the method of mixtures, for example, the result cannot be better than 1 part in 100 (the accuracy of the simple thermometer used), regardless of the fact that weighings may have been made on a balance with a degree of accuracy of 1 part in 1,000—or more.

Methods which are based on the balancing of two effects will give a higher degree of accuracy than methods which rely on the explicit measurement of the effect. This is because certain of the errors tend to balance each other out, and also because, in a null method, positions of balance are found more certainly and accurately than the direct measurement of a scale reading.

## 4. Calculation of the Results

The results of an experiment should be calculated to a degree of accuracy merited by the observations. Generally, four-figure mathematical tables will give the results with sufficient accuracy. Such tables can be relied upon to an accuracy of about 1 in 2,500, but where sensitive apparatus is used, as for example in experiments with a good spectrometer, it may be necessary to use five-figure or even seven-figure logarithms to obtain the necessary degree of accuracy. Slide rules (10-inch) cannot in general be relied upon to a greater degree than 1 in 500 and should never be used in working out the result of an accurate experiment.

In the final statement of the result no more significant figures should be given than is justified by the observations, and also by the extent to which the theory is applicable in the particular circumstances.

## 5. Graphs

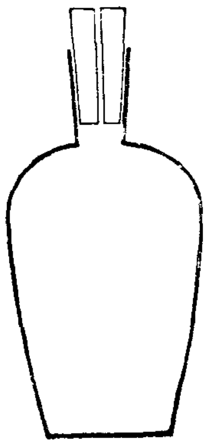
Wherever possible, the results of an experiment should be presented in graphical form. Not only does a graph provide the best means of averaging a set of observations, but the dependence between the quantities is clearly shown. In plotting the results, the dependent variable should be plotted as ordinates on the $y$-axis, and the independent variable as abscissae on the $x$-axis. The scale used should be a convenient one for arithmetical work, and should be sufficiently extensive for the graph to occupy a wide sweep of the space available. On the other hand, too large a scale will tend to accentuate the errors of observation, and obscure the relationship between the two quantities. Each point on the graph is an actual observation, and should be made to stand out by surrounding it with a small circle. The departure of the point from the final curve is a measure of the experimental error in that observation.

Wherever possible the straight-line graph should be used. This is more accurately drawn, and deductions from such a graph more reliable, than with curved graphs. If the relationship between the two quantities is not a simple linear one, it is usually possible to obtain the straight-line form by plotting powers of one or other (or both) of the quantities. For example, in the case of the simple pendulum, a plot of $t$ against $l$ results in a parabolic graph, but $t^2$ against $l$ becomes a straight line; and in the bifilar suspension, $t$ against $l$ produces a rectangular hyperbola, which can be converted into the straight-line form by plotting $t$ against $\frac{1}{l}$. The use of logarithmic scales is another means of obtaining the straight-line form in many cases. This method can often be used to obtain the empirical relation between two quantities.

Observations should be taken over as wide a range as possible, and the graph confined to the limits of the observations. In taking gradients, the full range of the graph (if a straight line) should be used; and for extrapolated values, the graph is continued as a broken line, and the result qualified by the statement "extrapolated value," indicating that it is outside the limits of actual observation.

Experiment 1

## DETERMINATION OF THE SPECIFIC GRAVITY OF (*a*) TURPENTINE, (*b*) SAND, (*c*) SALT, USING A SPECIFIC-GRAVITY BOTTLE



### Apparatus

Specific-gravity bottle, methylated spirits, turpentine, sand, salt, balance and weights.

### Method

All the above specific gravities can be found from the following seven weighings :—

First the specific-gravity bottle is thoroughly cleaned and dried. This can best be done by rinsing out well with methylated spirits and then drying by a stream of warm air from a heated tube connected to foot-bellows. The weight of the empty bottle (and stopper) is found. The bottle is now filled with water and reweighed. After emptying out the water and re-drying, the bottle is filled with turpentine and again weighed. The turpentine is emptied out, and the bottle again cleaned and dried. It is then filled about one-third full of sand and weighed. It is now filled with water, vigorously shaken to remove all air bubbles, topped up with water, dried on the outside and weighed. After emptying out the contents and cleaning and drying, it is weighed about one-third full of salt. Since salt is insoluble in turpentine, sufficient turpentine is now added to fill the bottle completely (after removing air bubbles), and after drying, the bottle is again weighed.

**Theory and Results**

| | | | | | | |
|---|---|---|---|---|---|---|
| Weight of | specific-gravity | bottle | empty ($w_1$) | | = | gm. |
| ,, | ,, | ,, | ,, | full of water ($w_2$) | = | gm. |
| ,, | ,, | ,, | ,, | ,, turpentine ($w_3$) | = | gm. |
| ,, | ,, | ,, | ,, | + sand ($w_4$) | = | gm. |
| ,, | ,, | ,, | ,, | + sand + water ($w_5$) | = | gm. |
| ,, | ,, | ,, | ,, | + salt ($w_6$) | = | gm. |
| ,, | ,, | ,, | ,, | + salt + turpentine ($w_7$) | = | gm. |

(*a*) Specific gravity of turpentine

$$= \frac{\text{weight of a given volume of turpentine}}{\text{weight of an equal volume of water}}$$

$$= \frac{w_3 - w_1}{w_2 - w_1} = \underline{\qquad\qquad}$$

(*b*) Specific gravity of sand

$$= \frac{\text{weight of a given volume of sand}}{\text{weight of an equal volume of water}}$$

$$= \frac{w_4 - w_1}{w_2 - w_1 - (w_5 - w_4)} = \underline{\qquad\qquad}$$

(*c*) Specific gravity of salt

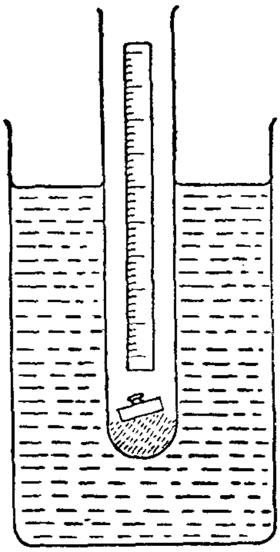
$$= \frac{\text{weight of a given volume of salt}}{\text{weight of an equal volume of water}}$$

$$= \frac{\text{weight of a given volume of salt}}{\text{weight of an equal volume of turpentine}} \times \frac{\text{weight of an equal volume of turpentine}}{\text{weight of an equal volume of water}}$$

$$= \frac{w_6 - w_1}{w_3 - w_1 - (w_7 - w_6)} \times \frac{w_3 - w_1}{w_2 - w_1} = \underline{\qquad\qquad}$$

EXPERIMENT 2

# DETERMINATION OF THE DENSITY OF A LIQUID USING A LOADED TEST-TUBE



## Apparatus

A wide test-tube fitted with a mm. scale (e.g. by inserting a strip of mm. graph paper, marked at cm. intervals from the bottom, inside it), a deep beaker, some lead shot, set of weights, callipers, methylated spirits or other liquid.

## Method

Sufficient lead shot is placed in the tube to make it float vertically in the liquid, and the level ($x_0$) at which it floats is noted. Weights are then placed in the tube and in each case the reading ($x$) of the scale against the liquid level is noted. A graph is then drawn of depth of immersion ($d$) $= x - x_0$ against additional load. The external radius of the tube is found by using callipers, taking four readings.

## Results

Calliper readings: , , , , mean diameter = cm.

$\therefore$ radius ($r$) = cm.

| Additional load | Scale readings $x$ | Depth of immersion $d = x - x_0$ |
|---|---|---|
| $0$ | $x_0$ | $0$ |
| | | |

From graph—

$AB =$ cm.

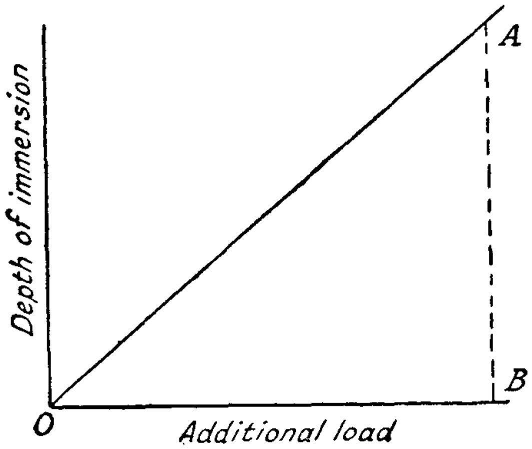
$OB =$ gm. wt.

$$\therefore \rho = \frac{OB}{\pi r^2 . AB}$$

$=$ ________ gm./c.c.

### Theory

Let the radius of the tube be $r$ cm., and let the mean depth of immersion in cm. per gm. be $n$ $\left(\text{from the graph } n = \frac{AB}{OB}\right)$. Then volume of liquid displaced by the addition of 1 gm. wt.




to the tube $= \pi r^2 n$ c.c. If the density of the liquid is $\rho$ gm./c.c., then the weight of the liquid displaced

$$= \pi r^2 n \rho \text{ gm.}$$

Hence by Archimedes' principle

$$\pi r^2 n \rho = 1$$

or

$$\rho = \frac{1}{\pi r^2 n} = \frac{OB}{\pi r^2 . AB} \text{ gm./c.c.}$$

### Notes

(1) If the loaded tube is immersed a further distance $x$ below the equilibrium position, a restoring force will be called into play of magnitude $\frac{x}{n}g$ dynes. The tube will execute vertical oscillations on being released, the equation of motion of the tube being—

$$M\ddot{x} = -\frac{g}{n}.x \text{ (where } M = \text{weight of tube} + \text{added load} + \text{weight of lead shot)}$$

i.e. $\ddot{x} + \frac{g}{Mn}.x = 0$. The motion is thus simple harmonic and the periodic time $T$ is—

$$T = 2\pi\sqrt{\frac{Mn}{g}}.$$

A value of $g$ can thus be found by timing, say, 5–10 oscillations from which to obtain $T$ and using the value of $n$ from the graph of the above experiment.

(2) The densities of two liquids may be compared, using the loaded test-tube as follows:

(*a*) by comparing the gradients of the graphs giving depth of immersion against load, thus

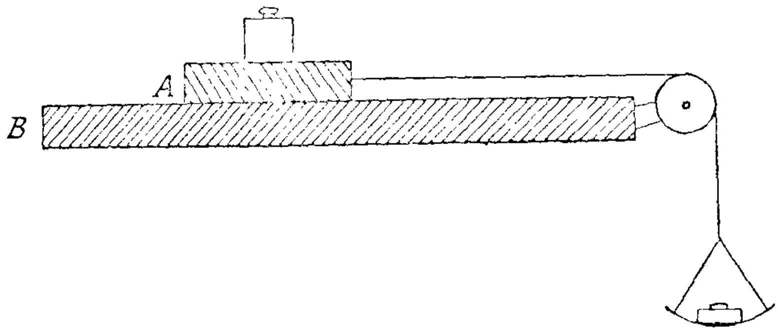
$$\frac{\rho_1}{\rho_2} = \frac{n_2}{n_1} \text{ (see above).}$$

(*b*) by comparing the times of oscillation of the tube in the two liquids with the same load in the tube in each case, thus

$$\frac{\rho_1}{\rho_2} = \frac{T_2^2}{T_1^2} \text{ (see above).}$$

EXPERIMENT 3

## DETERMINATION OF THE COEFFICIENT OF STATICAL FRICTION FOR TWO SURFACES




### Apparatus

A block of wood $A$ with smooth under surface slides over a smooth base-board $B$ which is clamped to the bench and provided with a pulley at one end. A string is attached to the block and passes over the pulley to support a scale-pan and weights. A set of weights for loading $A$. Spirit level.

### Method

The baseboard is first tested with a spirit-level and, if necessary, is made truly horizontal by inserting thin wedges below it. A known load ($W$) is placed on the sliding block, and weights ($w$) added to the scale-pan until the block just starts to move. The load is taken off the pan, and the experiment again repeated, commencing with the block in the same position as before. The whole experiment is repeated 3 or 4 times and the mean value of $w$ obtained. This procedure is repeated with different loads on the block. The weights of the block ($W'$) and the scale-pan ($w'$) are then found, $W'$ being added to $W$ to obtain the normal reaction $N$, and $w'$ added to $w$ to obtain the limiting frictional force $F$. A graph of $F$ against $N$ is then plotted, from which the coefficient of friction is found.

### Results

Weight of block ($W'$) = gm.

Weight of scale-pan ($w'$) = gm.

| Load $W$ | Normal Reaction $N = W + W'$ | Values of $w$ | Mean $w$ | Frictional force $F = w + w'$ |
|---|---|---|---|---|
| | | | | |

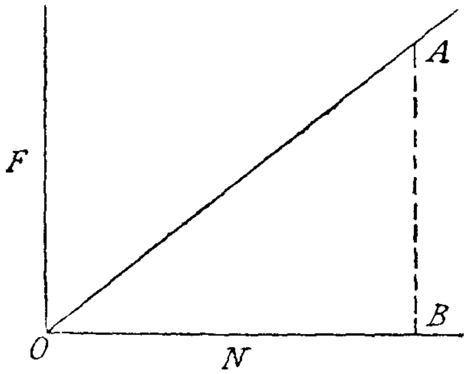
From the graph $AB =$ , $OB =$

$$\therefore \mu = \frac{AB}{OB} = \underline{\qquad\qquad}$$

### Theory

A small horizontal force applied to a body on a plane will not in general produce motion. This is due to the fact that at the surface of contact an equal and opposing force is called into play. This is known as the force of friction. If the applied force is gradually increased to a certain value, the body will just begin to move, the frictional force having increased to a limit known as the limiting friction. The laws of statical friction have been studied experimentally and may be stated as follows:

1. Before limiting friction is reached, the magnitude and direction of the frictional forces are such as just to preserve equilibrium.
2. The limiting friction between two surfaces is proportional to the normal reaction between them.
3. Friction is independent of the area of the two surfaces in contact.




From law (2) if $F$ is the frictional force and $N$ the normal reaction,

$\frac{F}{N}$ = constant ($\mu$)—known as the *coefficient of friction.*

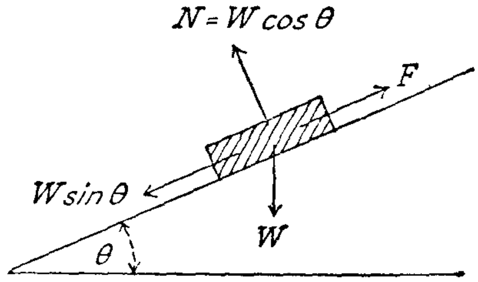
$\mu$ can be found by plotting $F$ against $N$ and taking the gradient of the graph, thus—

$$\mu = \frac{AB}{OB}.$$

### Notes

(1) The value of $\mu$ can also be obtained by placing the block on an inclined plane. As the inclination of the plane to the horizontal is increased, the block will just be on the point of moving at some angle $\theta$. The limiting frictional force is then clearly $= W \sin\theta$ and the normal reaction of the plane $= W \cos\theta$.

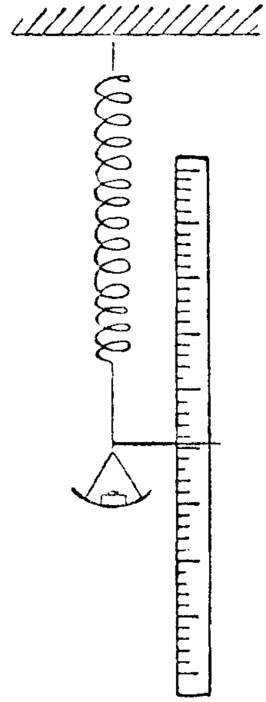
Hence $\mu = \frac{F}{N} = \frac{W \sin\theta}{W \cos\theta} = \tan\theta.$




(2) The angle between the normal reaction and the resultant reaction (compounded from the normal reaction and the frictional force) is called the angle of friction $\lambda$. Clearly, then, $\tan\lambda = \frac{F}{N}$, and in the experiment above, the limiting inclination of the plane = angle of friction.

(3) When motion between two surfaces takes place, it can be shown that the frictional force (dynamic friction) called into play is less than the limiting (statical) friction, and that it is independent of the relative velocity of the two surfaces, provided that this is not too large.

# EXPERIMENTS WITH A SPIRAL SPRING



### Apparatus

Spiral spring to which a light pointer is attached by plasticine at its lower end, rigid stand and clamp, metre rule, scale-pan and weights, stop-watch.

### Method

*Experiment (1). To verify Hooke's law, and to find the extension per gm. of added load.*

The spring, with scale-pan attached, is firmly clamped and the metre scale placed vertically so that the pointer moves lightly over it. Loads are added to the scale-pan and the corresponding extensions of the spring are noted. The scale readings are also taken when unloading the spring, and the mean extension thus obtained. A graph of extension against load is plotted from which the extension ($n$) per unit load is found.

*Experiment (2). To determine the acceleration of gravity and the effective mass ($m$) of the spring.*

A load is added to the pan which is set in vertical vibration by giving it a small additional displacement. The periodic time ($T$) is obtained by timing 20 vibrations. This is repeated with different loads and a graph of $T^2$ against load is plotted, from which $g$ and $m$ are found.

*Note.*—The mass of the scale-pan should be included in the load in this experiment.

### Results

*Experiment 1*

| Load $M$ | Extensions | | Mean extension |
|---|---|---|---|
| | Load increasing | Load decreasing | |
| | | | |

*Experiment 2*

| $M$ | Time for 20 vibrations | $T$ | $T^2$ |
|---|---|---|---|
| | | | |

From graph (1)

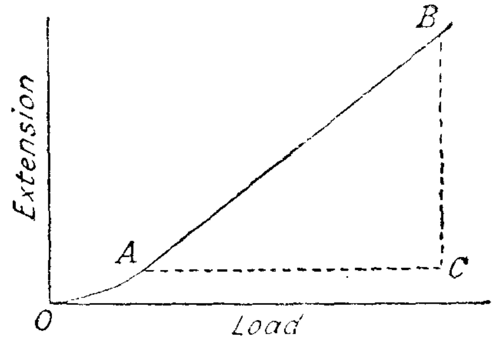
$$n = \frac{BC}{AC} = \_\_\_\_\_\_\_\_ \text{ cm./gm.}$$

$$g = 4\pi^2 n \frac{M}{T^2} \text{ from graph (2) } \frac{M}{T^2} = \frac{AC}{BC}$$

$$\therefore g = 4\pi^2 n \frac{AC}{BC} \_\_\_\_\_\_\_\_ \text{ cm./sec.}^2$$

also from graph (2) $m = OD =$ ________ gm.

**Theory**



(1)

A spiral spring subject to extension by an applied load conforms to Hooke's law, which states that the stress is proportional to the strain, i.e. the load is proportional to the extension it produces. If a graph is drawn, after the initial loading, where some force is required to separate the turns of the spring which are pressed against each other, a straight line is obtained of extension against load. From this portion, the extension ($n$) in cm. per gm. weight of load can be obtained from the gradient:

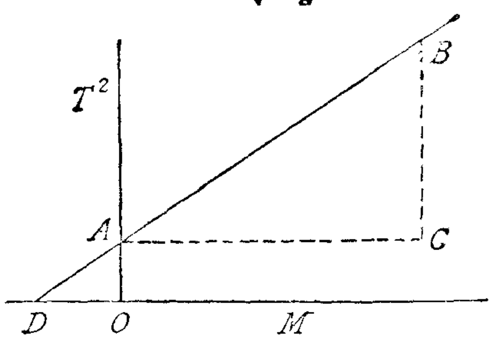
$$n = \frac{BC}{AC} \text{ (graph (1) ).}$$

If, now, a mass $M$ is attached to the spring, and the spring be extended a further distance $x$ cm., a restoring force $-\frac{x}{n}g$ dynes is called into play. The spring on being released executes vertical oscillations, the equation of motion of the mass being—

$$M\ddot{x} = -\frac{x}{n}\cdot g \qquad \text{or} \qquad \ddot{x} + \frac{g}{Mn}x = 0$$

The motion is thus simple harmonic and the periodic time $T$ is—

$$T = 2\pi\sqrt{\frac{Mn}{g}}.$$



(2)

The above analysis assumes the spring to be weightless. The load $M$ must be increased by an amount $m$ equal to the effective mass of the spring.

Then
$$T = 2\pi\sqrt{\frac{(M+m)n}{g}}.$$

If a graph of $T^2$ against $M$ is drawn, a straight line is obtained from which $g$ and $m$ can be found. The gradient of this line $= \frac{T^2}{M} = \frac{4\pi^2 n}{g} = \frac{BC}{AC}$ (graph (2) ), i.e., $g = \frac{4\pi^2 nAC}{BC}$.

The intercept $OD$ on the axis of $M$ gives the effective mass ($m$) of the spring.

**Note**

From considerations of the kinetic energy of the vibrating spring it can be shown that $m = \frac{1}{3}$ mass of spring.

# DETERMINATION OF THE SENSITIVITY OF A BALANCE, AND TO OBTAIN A VALUE OF "$g$" FROM THE TIME OF SWING

## Apparatus

A good balance, set of weights, stop-watch.

## Method

The balance is adjusted so that the pointer hangs centrally with respect to the scale. A small displacement is given to the unloaded balance, and the time for 20 swings is taken from which the periodic time $T_1$ is obtained. A 100-gm. weight is now placed in each pan, and the new periodic time $T_2$ found.

The sensitivity of the balance is now found by removing the 100-gm. weights, and placing a milligramme weight in one of the pans. The rest position ($x$ cm. from the centre scale division) is observed. The length ($y$ cm.) of the pointer is measured, and the angular deflection $\delta\theta$ of the balance beam is obtained from these two measurements.

## Results

Balance unloaded: Time for 20 swings = sec. $\therefore T_1 =$ sec.

100 gm. in each pan: Time for 20 swings = sec. $\therefore T_2 =$ sec.

Length of balance beam between pan pivots = cm. $\therefore l =$ cm.

Scale deflection ($x$) for 1 mg. ($\delta m$) difference in weight between the pans = cm.

Length of pointer ($y$) = cm.

i.e. $$\delta\theta = \frac{x}{y} =$$

$\therefore$ sensitivity $$= \frac{\delta\theta}{\delta m} = \frac{1000x}{y} = \_\_\_\_\_\_\_\_\_\_$$

$$g = \frac{8\pi^2 ml}{(T_2^2 - T_1^2)} \cdot \frac{\delta\theta}{\delta m} = \_\_\_\_\_\_\_\_\_\_ \text{ cm./sec.}^2$$

## Theory

When the beam of a balance is displaced through a small angle, a restoring couple is called into play proportional to the angle. If the moment of inertia of an unloaded balance is $I$, then on being displaced through an angle $\theta$, the equation of motion of the beam is $I\ddot{\theta} = -c\theta$, where $c$ is the restoring couple per unit angular displacement. The motion is thus simple harmonic, and the periodic time $T_1$ is

$$T_1 = 2\pi\sqrt{\frac{I}{c}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If, now, each pan is loaded with $m$ gm., and the half-length of the balance beam is $l$, the moment of inertia of the balance is now $I + 2ml^2$ and the new periodic time $T_2$ is

$$T_2 = 2\pi\sqrt{\frac{I + 2ml^2}{c}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From (1) and (2)

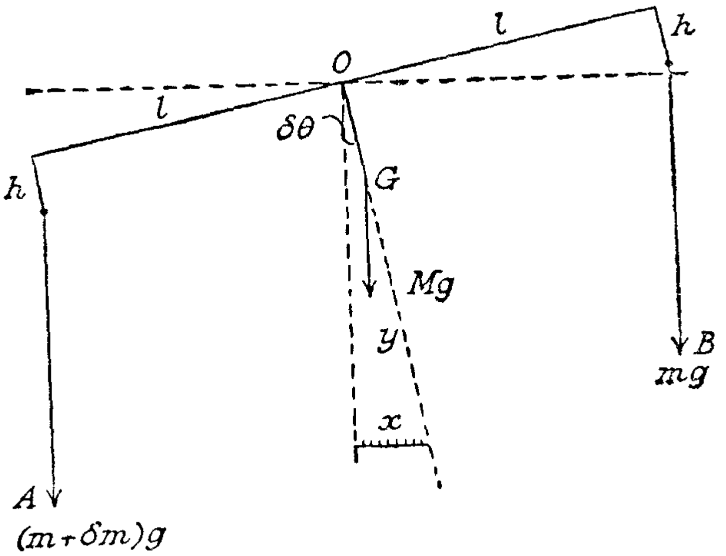
$$c = \frac{8\pi^2 ml^2}{T_2^2 - T_1^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

To find $c$ place a small weight ($\delta m$) in one of the pans, and observe the angular deflection ($\delta\theta$) of the beam. Then the deflecting couple is

$$\delta mgl = c\delta\theta \text{ or } c = \frac{\delta m}{\delta\theta}\cdot gl \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From (3) and (4)

$$g = \frac{8\pi^2 ml\,\delta\theta}{(T_2^2 - T_1^2)\delta m}.$$




### Note

*Sensitivity of a balance.* Let the mass of each pan and suspension be $m$, and the mass of the beam $M$. Let the C.G. of the beam be $d$ cm. below the point of suspension $O$, and let the pans be supported at a distance of $h$ cm. below the horizontal through $O$. If an additional small weight $\delta m$ placed in pan $A$ produces a deflection of $\delta\theta$, then taking moments about $O$—

$$(m + \delta m)g(l\cos\delta\theta - h\sin\delta\theta)$$

$$= mg(l\cos\delta\theta + h\sin\delta\theta) + Mgd\sin\delta\theta$$

$$\text{i.e. } -2mgh\sin\delta\theta + \delta mg(l\cos\delta\theta - h\sin\delta\theta) = Mgd\sin\delta\theta,$$

since $\delta\theta$ is small, we may take $\sin\delta\theta = \delta\theta$ and $\cos\delta\theta = 1$. Hence ignoring the second-order term:

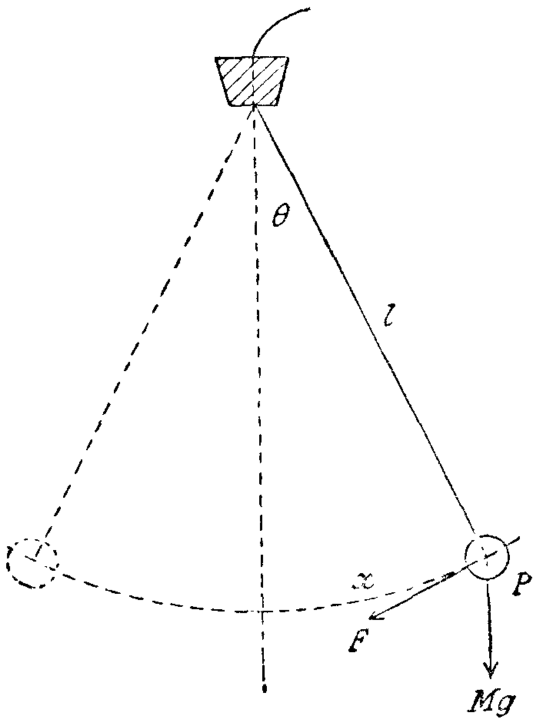
$$\delta mgl = (Md + 2mh)g\,\delta\theta$$

i.e.

$$\text{sensitivity} = \frac{\delta\theta}{\delta m} = \frac{l}{Md + 2mh}.$$

From this it is seen that the sensitivity depends (amongst other things) upon the total mass in the pans, but in practice $h$ is very small and hence increasing loads have little effect on the sensitivity of the balance. Accordingly it is justifiable for a first approximation to consider $c$ to be constant in the equations used for evaluating $g$, as was done in the method given above.

# DETERMINATION OF THE ACCELERATION OF GRAVITY BY MEANS OF A SIMPLE PENDULUM




## Apparatus

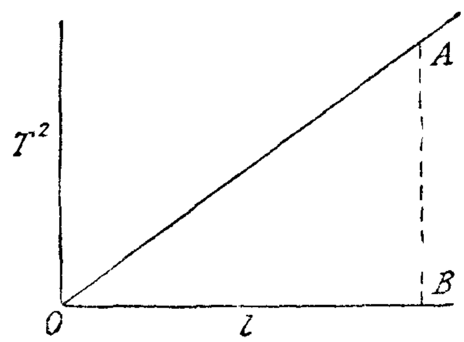
Small pendulum bob of lead or brass tied to a length (1 metre) of cotton threaded through a supporting cork. Retort-stand and clamp, metre rule, stop-watch.

## Method

The cork is securely clamped in position with the pendulum overhanging the bench. The length of the pendulum (measured from the point where the cotton emerges from the cork to the centre of the bob) is adjusted by drawing the cotton through the cork. The free end is secured on the clamp, and the pendulum given a small displacement. The time of 20 swings—measured against a fixed mark on the bench—is taken, and the periodic time ($T$) found. (The reading must be rejected if the swings become elliptical.) This is repeated with different lengths ($l$) of the pendulum, and a graph is drawn between $T^2$ and $l$ from which an average value of $\frac{T^2}{l}$ is obtained to determine $g$.

## Results

| $l$ cm. | Time for 20 swings | Time ($T$) per swing |
|---|---|---|
| | | |




$$g = \frac{4\pi^2 l}{T^2}$$

from the graph—

$$\frac{l}{T^2} = \frac{OB}{AB}$$

i.e. $g = 4\pi^2 \frac{OB}{AB}$

$= \underline{\hspace{4cm}}$ cm./sec.$^2$.

### Theory

The ideal simple pendulum consists of a point mass suspended by a weightless string. For a small angular displacement $\theta$ the restoring force acting on the point mass at $P$ along the arc $(x)$ is $F = -Mg\sin\theta = -Mg\theta$ (since $\theta$ is small) $= -Mg\frac{x}{l}$. Hence the equation of motion of $P$ is—

$$-\frac{Mgx}{l} = M\ddot{x}$$

or

$$\ddot{x} + \frac{g}{l}.x = 0.$$

The motion is thus simple harmonic, and the periodic time $T$ is—

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

### Notes

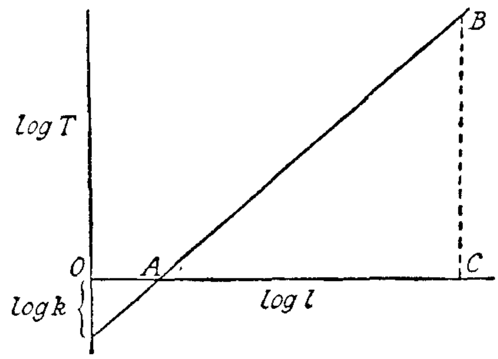
(1) It will be noted that the periodic time is independent of the mass of the bob and the amplitude of swing (if this is small). These facts can be verified experimentally by the student.

(2) The determination of $g$ from the time of swing of a simple pendulum is unsatisfactory for the following reasons:

- (a) the ideal of a point mass cannot be realized in practice,
- (b) the string is not weightless,
- (c) during the motion of $P$ the string is subject to varying tensions, and consequently errors are introduced due to the accompanying flexing of the string.

To overcome these objections, rigid pendulums must be used (see Experiments 7 and 8).

(3) The empirical determination of the formula for the simple pendulum provides an instructive experiment for the student.



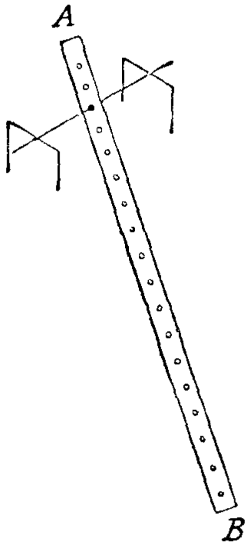

Let $T = kl^n$, then taking logs—

$$\log T = n\log l + \log k.$$

The value of $n$ can be found from the gradient of the straight-line graph obtained by plotting $\log T$ against $\log l$. Thus $n = \frac{BC}{AC}$. $k$ can be found from the intercept ($\log k$) on the $\log T$ axis.

EXPERIMENT 7

# DETERMINATION OF THE ACCELERATION OF GRAVITY BY MEANS OF A COMPOUND PENDULUM




## Apparatus

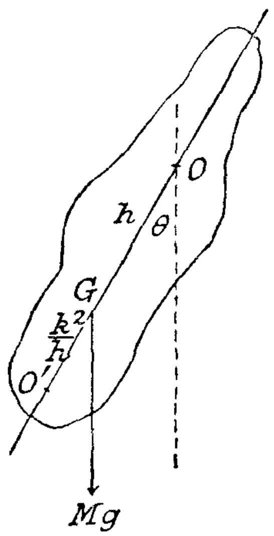
The usual pendulum for this experiment is a metal bar about a metre long drilled with holes at regular intervals and supported by a knife edge through these holes. If this is not available a metre rule can be used with small holes drilled at 4-cm. intervals. A knitting-needle passes through these holes, and supports the rule on two rigidly held razor blade edges. Stop-watch.

## Method

The needle is first inserted through the hole nearest the end $A$ of the bar, and the time for 20 vibrations of small amplitude is taken by the stop-watch. This is repeated for each point of suspension along the rule. From the observations made a graph is plotted of the periodic time ($T$) against the distance ($d$) of the suspension from the end $A$ of the rule. By drawing horizontal lines on this graph through given values of $T$, the corresponding mean values of the length ($l$) of the Simple Equivalent Pendulum can be obtained. An average value of $\frac{l}{T^2}$ can be obtained from these results for use in computing $g$.

## Theory

The diagram represents a rigid body suspended by a horizontal axis through $O$. On being displaced through a small angle $\theta$ from the vertical position a restoring couple




$-Mgh \sin\theta = -Mgh\theta$ (since $\theta$ is small) is called into play. The equation of motion of the body is thus—

$-Mgh\theta = I\ddot{\theta}$ (where $I$ is the moment of inertia about the axis through $O$).

The motion is thus simple harmonic, and the periodic time ($T$) is—

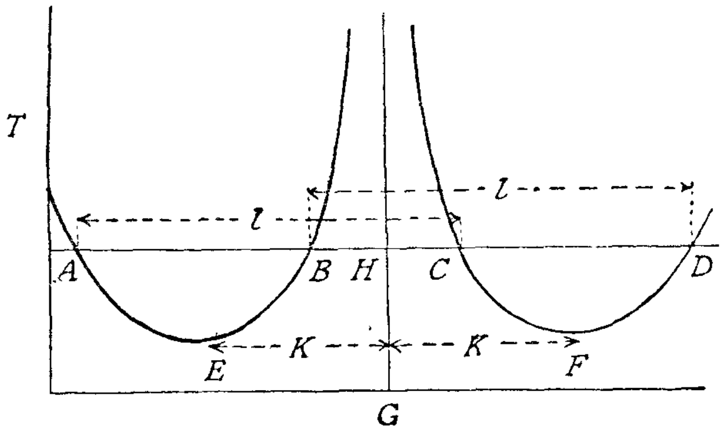
$$T = 2\pi\sqrt{\frac{I}{Mgh}}.$$

Writing $I_G$ for the moment of inertia about the C. of G., then $I = I_G + Mh^2$ (by theorem of parallel axes), and $I_G = Mk^2$, where $k$ is the radius of gyration about the C.G.

i.e. $$T = 2\pi\sqrt{\frac{k^2 + h^2}{gh}}.$$

Since the periodic time of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$ (see p. 23), the period of the rigid body is the same as that of a simple pendulum of length $l = h + \frac{k^2}{h}$. This is known as the length of the Simple Equivalent Pendulum. The expression for $l$ may be written as a quadratic in $h$, thus: $h^2 - hl + k^2 = 0$. This gives two values of $h$

($h_1$ and $h_2$) for which the body has equal times of vibration. From the theory of quadratic equations, $h_1 + h_2 = l$ and $h_1h_2 = k^2$. Thus if a distance $\frac{k^2}{h_1}$ is measured along the axis from $G$ on the side remote from $O$, a point $O'$ (the centre of oscillation) is obtained, and the distance $OO'$ = length of the S.E.P. The periodic time about $O'$ is clearly the same as that about $O$, i.e. the centres of suspension and oscillation are interchangeable.




A graph of $T$ against $d$ will be symmetrical about a line through the C.G. (for which $T$ is infinite) as shown, and a horizontal line drawn through a given value of $T$ will cut the graph in four points. The length $l$ of the S.E.P. for this value of $T$ will be the distance from the 1st to the 3rd or the 2nd to the 4th of these points

then $$g = \frac{4\pi^2 l}{T^2}.$$

**Notes**

(1) The roots of the quadratic at the bottom of p. 24 are— $h = \frac{l}{2} \pm \frac{1}{2}\sqrt{l^2 - 4k^2}$.

The least value of $l$ for real roots is $2k$ when $h_1 = h_2 = \frac{l}{2} = k$ and the time is then a minimum $= 2\pi\sqrt{\frac{2k}{g}}$. The radius of gyration can be found directly from the graph, $k = \frac{EF}{2}$, or alternatively $k$ can be found from the relation $k = \sqrt{h_1h_2} = \sqrt{AH \times HC}$.

(2) Having found $k$ as above, the moment of inertia of the body about the C.G. can be determined, having obtained its mass by weighing, $I_G = Mk^2$.

(3) The radius of gyration (and hence the moment of inertia) of a rigid body can be obtained very readily by suspending it at a given distance ($h$) from its C.G. on the same axis as a simple pendulum. The length ($l$) of the simple pendulum is adjusted so that both pendulums swing together. Then $l = h + \frac{k^2}{h}$, from which $k$ can be found.

**Results**

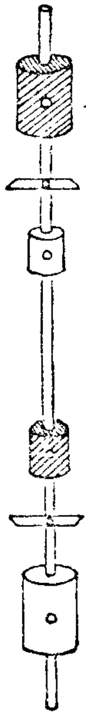
| $d$ | Time for 50 vibrations | $T$ |
|---|---|---|
| | | |

Results from graph:

| $T$ | $T^2$ | Mean S.E.P. $l$ | $\frac{l}{T^2}$ |
|---|---|---|---|
| | | | |
| | | Average = | |

$g$ = ________________ cm./sec.$^2$.

# DETERMINATION OF THE ACCELERATION OF GRAVITY BY MEANS OF A KATER PENDULUM




## Apparatus

Any of the standard forms of Kater pendulum, one of which is shown in the diagram. A brass rod is provided with two knife-edges $EE$ turned inwards to face each other. Beyond the knife-edges are two equal cylinders, one of boxwood ($A$) and the other of brass ($B$). Two smaller equal cylinders ($C$ of brass and $D$ of boxwood) are placed between the knife-edges, and are movable along the rod. (By using this symmetrical arrangement of weights about the midpoint of the rod, the effects of air resistance are eliminated.) Accurate stop-watch. Pivot with knife-edge.

## Method

The knife-edges $EE$ are fixed a given distance apart. The pendulum is then suspended from one knife-edge and the period of vibration ($T_1$) is found by timing 50 swings. It is then suspended by the other knife-edge, and by adjusting the cylinders $C$ and $D$, a periodic time ($T_2$) about this knife-edge is obtained as nearly equal to the first as possible. The pendulum is then returned to its first suspension and $T_1$ redetermined. It is again reversed and further adjustments made to obtain an equal time of swing. Proceeding in this manner, $T_2$ is obtained very nearly equal to $T_1$, after which $T_1$ and $T_2$ are accurately determined by timing at least 200 swings. The centre of gravity of the bar is now found by pivoting the bar on a sharp knife-edge, when $h_1$ and $h_2$ can be determined.

## Results

1st knife-edge: Time for swings = sec. $\therefore T_1 =$ sec.

2nd knife-edge: Time for swings = sec. $\therefore T_2 =$ sec.

Distance between knife-edges ($h_1 + h_2$) = cm.

Distance of 1st knife-edge from point of balance ($h_1$) = cm.

Distance of 2nd knife-edge from point of balance ($h_2$) = cm.

$\therefore g =$ ________________ cm./sec.$^2$

### Theory

If the movable cylinders are so adjusted that the periodic time $T$ is the same about the two knife-edges, then the distance $l$ between them is the length of the Simple Equivalent Pendulum, and hence—

$$g = \frac{4\pi^2 l}{t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Bessel showed that the accurate determination of $g$ did not require the lengthy process of getting the two periods exactly alike. It was sufficient if they were nearly equal, thus—

**Let** $$T_1 = 2\pi\sqrt{\frac{h_1^2 + k^2}{gh_1}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{h_2^2 + k^2}{gh_2}}$$

**then** $$\frac{gh_1T_1^2}{4\pi^2} = h_1^2 + k^2 \quad \text{and} \quad \frac{gh_2T_2^2}{4\pi^2} = h_2^2 + k^2$$

subtracting and rearranging—

$$\frac{4\pi^2}{g} = \frac{T_1^2h_1 - T_2^2h_2}{h_1^2 - h_2^2}$$

**or** $$\frac{4\pi^2}{g} = \frac{T_1^2 + T_2^2}{2(h_1 + h_2)} + \frac{T_1^2 - T_2^2}{2(h_1 - h_2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If the C.G. is much nearer to one set of knife-edges than the other, $h_1 - h_2$ is not small, and since $T_1$ is approximately equal to $T_2$, the second term is very small, and thus an approximate evaluation of $h_1 - h_2$ is sufficient.

### Notes

(1) For very accurate work the distance between the knife-edges is found by a comparator, and the values of $T_1$ and $T_2$ are found by the method of coincidences. This method consists in comparing the vibrations of the rigid pendulum with those of a seconds pendulum of a standard clock. The Kater pendulum is suspended in front of the standard pendulum with their lower ends on the same level. The pendulums are set vibrating and viewed through a telescope against a fixed point behind the far pendulum. If the periods are not the same they will get "out of step," and at a certain instant will pass the mark together when travelling in the same direction. This will not occur again until the pendulum has lost or gained a complete swing. If the standard pendulum makes $n$ complete swings of period $T$ and the Kater pendulum $(n + 1)$ of period $T_1$ in the same time, then—

$$Tn = T_1(n + 1)$$

**i.e.** $$T_1 = T\left(\frac{n}{n+1}\right) = T\left(1 + \frac{1}{n}\right)^{-1} = T\left(1 - \frac{1}{n} + \frac{1}{n^2} - \ldots\right)$$

with $n$ large $T_1 = T\left(1 - \frac{1}{n}\right)$ gives $T_1$ to a high degree of accuracy.

(2) A correction for finite swing of arc can be applied in precise determinations of $g$ as follows—

$$T_0 = T_1\left(1 - \frac{\alpha_1\alpha_2}{16}\right)$$

where $T_0$ is the corrected periodic time, $T_1$ is the observed periodic time, and $\alpha_1$ and $\alpha_2$ are the half-angles of swing in radians at the start and finish respectively.

EXPERIMENT 9

## APPROXIMATE DETERMINATION OF THE ACCELERATION OF GRAVITY BY MEANS OF A SPHERE ROLLING ON A CONCAVE SURFACE

### Apparatus

Large concave mirror or watch-glass firmly embedded in plasticine in a shallow box, small steel ball-bearings, vernier callipers or screw gauge, stop-watch, spherometer.

### Method

The ball-bearings are first carefully cleaned of all oil and grease, and the mirror is wiped to remove any dust. A ball-bearing is placed near the edge of the mirror and released, and the time of as many oscillations as possible is recorded. This is repeated several times, and from the observations the mean periodic time is obtained. The radius of the ball-bearing is taken with the vernier callipers, and the radius of the concave surface is obtained, using a spherometer (see p. 68) if the surface is unsilvered—otherwise by optical means.

### Results

Times for          oscillations :     ,     ,     ,     ,     sec.

Mean time =          sec.

∴ Periodic time ($T$) =          sec.

Calliper readings of the diameter of sphere :     ,     ,     ,     ,

Mean diameter =          cm.

Zero error = ±          cm.

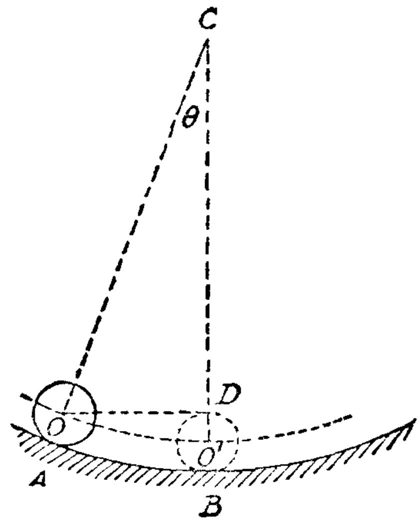
∴ true mean diameter =          cm.

i.e. radius ($a$) =          cm.

Radius of surface ($R$) =          cm.

$$g = \frac{28\pi^2(R - a)}{5T^2}$$

$$= \underline{\hspace{4cm}} \text{ cm./sec.}^2.$$




## Theory

A sphere of radius $a$, rolling over a concave surface of radius of curvature $R$, has a motion similar to the motion of the bob of a simple pendulum of length $(R - a)$, with the exception that the sphere rolls as it moves forward. Hence, in dealing with this problem, it is necessary to take into account the rotational energy of the sphere as well as its energy of translation.

In moving from its rest position at $A$ to some other position—say $B$—where its linear velocity is $v$, the sphere loses potential energy $= Mgh$ . . . (1) (where $h = O'D$).

The corresponding gain in kinetic energy

$$= \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2$$

$$= \tfrac{1}{2}.v^2\left(M + \frac{I}{a^2}\right) \qquad \text{since } \omega = \frac{v}{a}$$

$$= \tfrac{1}{2}.\tfrac{7}{5}Mv^2 \quad . \quad . \quad . \quad (2) \qquad \text{since } I = \tfrac{2}{5}Ma^2 \text{ for a solid sphere.}$$

Hence from (1) and (2)

$$v^2 = (\tfrac{5}{7}).2gh.$$

Now the corresponding velocity $v_1$ for a simple pendulum is given by—

$$v_1^2 = 2gh,$$

so that at all points on its path the velocity of the sphere is $\sqrt{\tfrac{5}{7}}$ of the velocity of the pendulum bob. Therefore to complete any given motion, the time of the sphere will be $\sqrt{\tfrac{7}{5}}$ as great as the time for a simple pendulum. Thus, since the periodic time of a simple pendulum is $2\pi\sqrt{\frac{l}{g}}$, where in this case $l = R - a$, the periodic time for the rolling sphere is—

$$T = 2\pi\sqrt{\frac{7(R-a)}{5g}}.$$

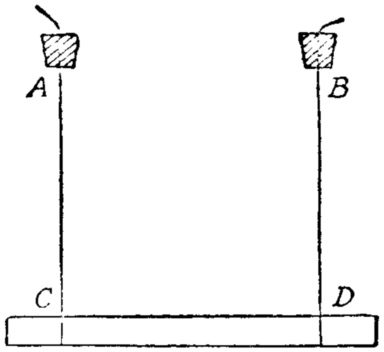
$$\therefore g = \frac{28\pi^2(R-a)}{5T^2}.$$

## Note

Alternatively, if a value of $g$ is assumed, the above arrangement can be used as a dynamical spherometer to determine the radius of curvature of a concave spherical surface.

EXPERIMENT 10

## DETERMINATION OF MOMENT OF INERTIA USING THE BIFILAR SUSPENSION




### Apparatus

Two heavy stands and clamps, two threaded corks, metre rule, brass rod, stop-watch.

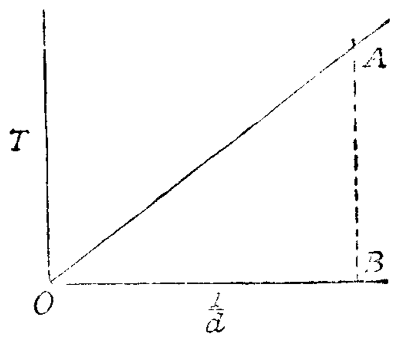
### Method

Pass the brass rod through two loops made on the ends of the two lengths of cotton passing through the corks. Firmly clamp the corks in two heavy stands. Arrange the threads at some distance $d$ apart, and adjust the lengths of the threads to some suitable length $l$. Tie off the loose ends of the threads on the clamp, and give the rod a small angular displacement about a vertical axis. Find the periodic time ($T$) by timing 20 vibrations. Repeat with the threads at varying distances apart, and plot a graph of $T$ against $\frac{1}{d}$ from which to obtain an average value of $Td$. Measure $l$ and find the mass $M$ of the rod by weighing.

### Results

$M =$ gm.
$l =$ cm.

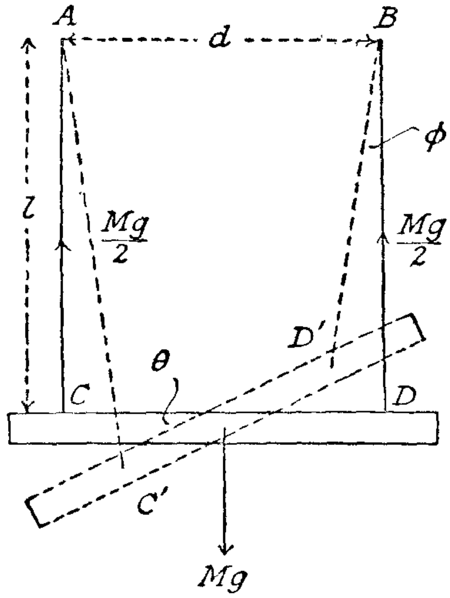
| $d$ | Time for 50 vibrations | $T$ |
|---|---|---|
| | | |




from the graph $Td = \frac{AB}{OB}$.

$$I = \frac{Mg}{16\pi^2 l}\cdot(Td)^2$$

$$= \frac{Mg}{16\pi^2 l}\left(\frac{AB}{OB}\right)^2 = \underline{\qquad\qquad\qquad} \text{ gm.-cm.}^2$$




**Theory**

Let a body of mass $M$ and moment of inertia $I$ be suspended by two parallel threads $AC$ and $BD$ $l$ cm. long and $d$ cm. apart. Then the tension in each of these threads will be $\frac{Mg}{2}$. Give the system a small angular displacement $\theta$ about a central axis, and let $\phi$ be the corresponding inclination of the strings to the vertical. Since both angles are small—

$$l\phi = \frac{d}{2}\theta.$$

The components of the tensions in the strings producing restoring forces at $C'$ and $D'$ are of magnitude $\frac{Mg}{2}\sin\phi = \frac{Mg}{2}\phi$ (since $\phi$ is small)

$$= \frac{Mgd\theta}{4l}.$$

The restoring couple on the rod is thus—

$$-\frac{Mgd}{4l}\theta . d$$

and the equation of motion of the rod is—

$$I\ddot{\theta} = -\frac{Mgd^2}{4l}.\theta \qquad \text{or} \quad \ddot{\theta} + \frac{Mgd^2}{4Il}.\theta = 0$$

the motion is thus simple harmonic of periodic time

$$T = 2\pi\sqrt{\frac{4Il}{Mgd^2}}. \quad \therefore I = \frac{Mgd^2T^2}{16\pi^2 l}$$

**Notes**

(1) Additionally, the value of $I$ may be obtained by keeping $d$ constant and varying $l$. The student should repeat the experiment under these conditions, and obtain the value of $I$ from the slope of the graph resulting from plotting $T^2$ as ordinates against $l$ as abscissae.

(2) If the threads are not parallel, but with distances between them of $d_1$ and $d_2$ at the ends, an analysis similar to the above shows that—

$$T = 2\pi\sqrt{\frac{4Il}{Mgd_1d_2}}.$$

(3) If two bodies of masses $M_1$ and $M_2$, and moments of inertia $I_1$ and $I_2$, have periodic times of $T_1$ and $T_2$ when suspended in turn by the same strings—

Then $$\frac{I_1}{I_2} = \frac{M_1T_1^2}{M_2T_2^2}.$$